SOAR

Teaching Frames® for Literacy

DEEP DIVE MANUAL

MATHEMATICAL DISCUSSIONS Manual

EPF Essential Practice Frames for teaching

SOAR
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DEEP DIVE MANUAL

MATHEMATICAL DISCUSSIONS in Elementary Classrooms

EPF *for teaching*

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Essential Teaching Practices and Practice Frames

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FOCUS QUESTIONS

- Which teaching practices are most essential for supporting implementation of College and Career Readiness Standards?
- In what ways do these essential teaching practices interconnect during instruction to support student learning?
- How can a corresponding set of classroom videos, vignettes, and instructional strategies be used to illustrate what the essential teaching practices look like when enacted in the classroom?

Essential Teaching Practices

The expectations for teachers as reflective practitioners when implementing new College and Career Readiness Standards* are plain to see. But where can educational professionals find answers to questions like these?

- What does the practice of a particular teacher look like in engaging and supporting all students in learning?
- How do these practices change as they emerge across the continuum of teaching?
- What tools can support development of these essential practices through sustained, reflective analysis and professional development?

Because we believe that questions like these are at the heart of successful CCSS Mathematics implementation, our team of educational researchers and practitioners have engaged in more than a decade of systematic research and development to identify the essential practices that teachers can use to integrate mathematics instruction and academic language development. More specifically, we identified research-based practices at the intersection of promoting academic-language development and disciplinary understanding through analyses of data from Delphi panel studies of expert consensus, [1] video observations of classroom instruction,[2] existing instructional practice protocols with established reliability and predictive validity, [3, 4] and an extensive review of the research literature on effective mathematics instruction. [5, 6, 7, 8, 9]

Five High-Impact Practices (see pages 10-11) emerged as having significant potential to enhance student learning as articulated in the CCSS Mathematics and state ELD standards. These practices, which are embedded within the SOAR Teaching Frames®, are the following:

* College and Career Readiness Standards include the Common Core State Standards (CCSS), the Next Generation Science Standards (NGSS), and state ELD Standards.

Mathematical Discussions: This practice focuses on structuring, strengthening, and supporting students' ability to engage in mathematical discussions. Mathematical discussions can consist of face-to-face interactions, online dialogues, and student-to-student number talks. Effective teachers build mathematical conversation skills and provide extended and supported opportunities for students to participate in mathematical discussions.[10, 11, 12, 13, 14]

Mathematical Thinking Processes: This practice focuses structuring, strengthening, and supporting mathematical thinking skills to deepen conceptual understanding as well as strengthen procedural fluency, strategic competence, and adaptive reasoning. Effective teachers provide explicit instruction about mathematical thinking processes. Effective teachers also provide extended and supported opportunities for students to engage in these thinking processes. Mathematical Thinking Processes include: representation, problem solving, reasoning and proof, and inquiry.[15, 16]

Communicating Mathematical Understanding: This practice focuses on structuring, strengthening, and supporting students' ability to communicate mathematical understanding through oral and written output. Examples include pictorial and symbolic representations, graphs, models, spreadsheets, and oral or written justifications as discussed in the CCSS. Effective teachers provide multiple and supported opportunities for students to communicate their understanding of mathematical concepts through oral and written output.[17, 18, 19]

Mathematical Perseverance: This practice focuses on structuring, strengthening, and supporting students' ability to persevere, which includes productive disposition and struggle, a growth mindset, and taking risks as part of the problem-solving process. Effective teachers intentionally build mathematical perseverance skills and create cognitively demanding tasks that are accessible, require long-term goal setting, and for which solution pathways are not readily apparent. [20, 21, 22, 23]

Acquisition of Foundational Numeracy Skills (TK-2 only): This practice focuses structuring, strengthening, and supporting students' understanding and working knowledge of counting and cardinality, numbers and operations, algebraic thinking, measurement, and geometry. The foundational skills being taught are aligned to the standards for the grade level being observed. [24]

Our research also suggests that these essential High-Impact Practices, while central to effective implementation of CCSS Mathematics, do not operate in isolation. Instead, effective teachers enact a set of dynamic instructional moves in support of the High-Impact Practices. We labeled this set of three instructional moves Cross-Cutting Practices.

Facilitating Acquisition of Academic Language: This practice focuses on structuring, strengthening, and supporting the acquisition and use of the academic language needed to participate in knowledge construction and mathematical tasks. [25, 26, 27]

Fostering Metacognition for Mathematical Learning: This practice focuses on the degree to which a teacher visibly enacts and deconstructs metacognitive processes and strategies that foster students' metacognitive knowledge. There are two aspects of metacognition: reflective processes, i.e., awareness of what we know, and self-regulation, i.e., taking action to address flaws or gaps in what we know. [28, 29]

Connecting and Guiding Mathematical Learning: This practice focuses on how effectively a teacher elicits student thinking and uses that to engage student preconceptions and guide mathematical learning throughout each task, lesson, and unit of instruction. [30, 31, 32]

Finally, in preparation for enactment of the High-Impact and Cross-Cutting Practices, teachers employ a Foundational Practice.

Designing Instruction for Mathematical Thinking and Understanding: This practice focuses on the design of lessons and learning tasks to promote mathematical learning and support the target High-Impact Practice. This practice also focuses on how the teacher establishes high expectations and fosters in all students the willingness to participate in intellectually rigorous tasks. [33, 34, 35, 36, 37]

SOAR Teaching Frames

To illustrate the interconnectedness of the practices, we organized the practices into teaching frames, each consisting of a different High-Impact Practice supported by the Cross-Cutting and Foundational Practices, which are common across all frames. We call the collective framework Essential Practice Frames (EPF). The *Mathematical Discussions Teaching Frame* can be found on pages 12-13.

Together, the **SOAR** Teaching Frames® for Mathematics (with associated materials and rubrics) form a teaching protocol for improving nine essential practices for grades TK-2 and eight essential practices for grades 3-12. This protocol breaks the practices down into their component parts, which we call elements, to allow teachers to fully understand the practices, enact them, and reflect on them to elevate their teaching. In addition, the protocol provides a common language so that teachers and coaches can provide constructive and strategic feedback to their peers. A clarification of the important terms for each practice is included.

The **SOAR** Teaching Frames® for Mathematics apply to the academic language and mathematical practices requirements of the CCSS Mathematics and ELD standards. Other teaching frames that have been developed as part of this ongoing work include **SOAR** Teaching Frames® for Literacy TK–2, **SOAR** Teaching Frames® for Literacy 3–12, and **SOAR** Teaching Frames® for English Language Development TK–12.

The Strategic Observation and Reflection (**SOAR**) Teaching Frames offer teachers, coaches, and administrators who are currently implementing new College and Career Readiness Standards a suite of tools that drive continuous improvement in teaching and learning. The tools can be used for a variety of purposes: (1) informal or guided self-assessment of teaching practices, (2) peer-to-peer collaboration within learning communities to improve teaching, (3) site- and district-based professional-growth initiatives, and (4) formative and summative teacher assessment.

Implementing the SOAR Protocol

We designed this protocol to be used in several ways. First, we believe that it will support teachers in improving their ability to integrate mathematics instruction and academic language development and support their implementation of the CCSS Mathematics and state ELD Standards. Second, we believe it will foster peer-to-peer collaboration and facilitate cycles of strategic observation and reflection that are essential to improving teaching and learning. Having identified the practices that are most predictive of student growth, our team has also developed a corresponding set of videos and materials to illustrate what these practices look like when enacted in the classroom. The **SOAR** team has developed a platform designed around the **SOAR** Teaching Frames® (soarpractices.org), which fulfills several functions:

- Helps teachers and administrators deepen their understanding of the **SOAR** practices by engaging in a series of online modules and accessing additional instructional strategies.
- Allows teachers and administrators to use **SOAR** for self-reflection and for formative and summative teacher assessment, by providing interactive activities to deepen understanding of the **SOAR** rubrics.
- Scaffolds professional-learning for teachers in different grade spans (TK–2, 3–8, and 9–12) and across content areas (ELA, social studies, science, and mathematics) by providing them access to additional video examples of the practices implemented in a range of classrooms.

Conclusion

This chapter describes essential instructional practices that drive student learning as articulated in the CCSS Mathematics and state ELD Standards, as well as a framework for how these essential practices are interconnected in an effective integration of mathematics instruction and academic language development. We are currently using the protocol and corresponding online support materials in professional-growth programs for teachers, coaches, and instructional leaders in partner districts and schools. Findings from this work demonstrate that the **SOAR** Teaching Frames® for Mathematics provide a suite of powerful tools for supporting leaders', coaches', and teachers' adoption and enactment of these practices in their contexts, and ultimately for improving mathematics outcomes for all students.

Footnotes

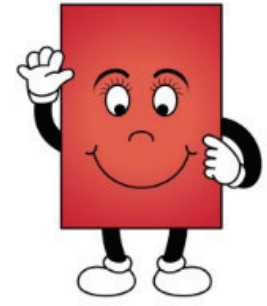
1. Pritchard, R., O'Hara, S., & Zwiers, J. (2016). Framing the teaching of academic language to English learners: A Delphi study of expert consensus. *TESOL Quarterly.* doi:10.1002/tesq.337.
2. O'Hara, S., Pritchard, R., & Zwiers, J. (2016). Academic language and literacy in every subject (ALLIES): A capacity building approach to supporting teachers in grades 4–8. In P. Proctor, A. Boardman, & E. Hiebert (Eds.), *English learners and emergent bilingualism in the common core era* (pp. 197-214). New York, NY: Guilford Press.
3. Bill & Melinda Gates Foundation. (2014). *Building trust in observations: A blueprint for improving systems to support great teaching*. Seattle, WA: Author.
4. Hill, H., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G., Sleep, L., & Ball, D. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, *26*, 430–511.
5. Blazar, D. (2015). Effective teaching in elementary mathematics: Identifying classroom practices that support student achievement, *Economics of Education Review*, 48, 16-29.
6. Boaler, J. (2015). *What's math got to do with it?: How teachers and parents can transform mathematics learning and inspire success*. New York, NY: Penguin.
7. Driscoll, M. J., DiMatteo, R. W., Nikula, J., & Egan, M. (2007). *Fostering geometric thinking: A guide for teachers, grades 5-10*. Portsmouth, NH: Heinemann.
8. Moschkovich, J. (2013). Principles and Guidelines for Equitable Mathematics Teaching Practices and Materials for English Language Learners. *Journal of Urban Mathematics Education*, 6(1), 45–57.
9. Pritchard, R., O'Hara, S., & Zwiers, J. (2014). Using new technologies to engage and support English learners in mathematics classrooms. In D. Polly (Ed.), *Cases on technology and common core mathematics standards* (pp. 145–163). Hershey, PA: IGI Global.
10. Zwiers, J., & Crawford, M. (2011). *Academic conversations*. Portland, ME: Stenhouse.
11. *Smith, Margaret Schwan. (2011). 5 practices for orchestrating productive mathematics discussions. Reston, VA: [Thousand Oaks, CA]: National Council of Teachers of Mathematics; Corwin.*
12. Kazemi, E., & Hintz, A. (2014). *Intentional talk: How to structure and lead productive mathematical discussions*. Portland, ME: Stenhouse Publishers.
13. Zwiers, J., O'Hara, S., & Pritchard, R. (2014d). Conversing to fortify literacy, language, and learning. *Voices from the Middle, 22*(1), 10–14.
14. Zwiers, J., O'Hara, S., & Pritchard, R. (2014a). *Common Core Standards in diverse classrooms: Essential practices for developing academic language and disciplinary literacy*. Portland, ME: Stenhouse.
15. English, L. D., & Gainsburg, J. (2016). Problem solving in a 21st century mathematics curriculum. In L. D. English and D. Kirshner (Eds.), In *Handbook of International Research in Mathematics Education* (3rd ed.) (Vol. 3, pp. 313-335). New York, NY: Taylor and Francis.
16. Chamberlin, M. T. (2005). Teachers' discussions of students' thinking: Meeting the challenge of attending to students' thinking. *Journal of Mathematics Teacher Education*, 8, 141-170.
17. Parrish, Sherry D. (2011). Number talks build numerical reasoning. *Teaching Children Mathematics,*18(3), 198-206.
18. Viseu, F., & Oliveira, I. B. (2012). Open-ended tasks in the promotion of classroom communication in mathematics. *International Electronic Journal of Elementary Education, 4*(2), 287–300.
19. Bicer, A., Capraro, R., & Capraro, M. (2013). Integrating writing into mathematics classrooms to increase students' problem-solving skills. *International Online Journal of Educational Sciences, 5*(2), 361-369.

20. Bass, H., & Ball, D. L. (2015). *Beyond "You can do it!": Developing mathematical perseverance in elementary school*. Retrieved from www.spencer.org.
21. Middleton, J. A., Tallman, M. A., Hatfield, N., & Davis, O. (2015). *Taking the severe out of perseverance: Strategies for building mathematical determination*. Retrieved from www.spencer.org.
22. Taylor, E. V. (2015). *Cultural considerations in support of mathematical perseverance: The role of context activation*. Retrieved from www.spencer.org.
23. Barnes, A. (2015). Improving children's perseverance in mathematical reasoning: Creating conditions for productive interplay between cognition and affect. *CERME 9* (pp. 1131-1138).
24. Reid, K. and Andrews, N. (2016). *Fostering Understanding of Early Numeracy Development*. https://research.acer.edu.au/monitoring_learning/29.
25. Heller, V. (2015). Academic discourse practices in action: Invoking discursive norms in mathematics and language lessons, *Linguistics and Education*, 31, 187-206.
26. Kibler, A., Walqui, A., & Bunch, G. (2015). Transformational opportunities: Language and literacy instruction for English language learners in the Common Core era in the United States. *TESOL Journal, 6*(1), 9–35. doi:10.10002/tesj.133.
27. Riccomini, P., Smith, G., Hughes, E. & Fries, K. (2015) The Language of Mathematics: The Importance of Teaching and Learning Mathematical Vocabulary, *Reading & Writing Quarterly*, 31(3), 235-252,
28. Ozsoy, G., Ataman, A. (2009). The effect of metacognitive strategy training on mathematical problem-solving achievement. *International Electronic Journal of Elementary Education*, 1, 67-82.
29. Bryce, D., & Whitebread, D. (2012). The development of metacognitive skills: evidence from observational analysis of young children's behaviour during problem-solving. *Metacognition and Learning, 7*, 197–217.
30. Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60*(4), 380–392.
31. Jacobs, V. R., & Empson, S. B. (2016). Responding to children's mathematical thinking in the moment: an emerging framework of teaching moves. *ZDM Mathematics Education*, 48(1–2).
32. M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 79–94). New York: Routledge.
33. Wiggins, G. (2012). Seven keys for effective feedback. *Feedback for Learning*, *70*(1), 10–16.
34. Brophy, J. (2010). *Motivating students to learn* (3rd ed.). New York, NY: Routledge.
35. Finley, T. (2015). The science behind classroom norming. *Edutopia.* Retrieved from http://www.edutopia.org/blog/establishing-classroom-norms-todd-finley.
36. Marzano, R., Yanoski, D., Hoegh, J., & Simms, J. (2013). *Using common core standards to enhance classroom instruction and assessment*. Centennial, CO: Marzano Research.
37. Wilson, H. W., Sztajn, P., Edington, C., & Myers, M. (2015). Teachers' uses of a learning trajectory in student-centered instructional practices. *Journal of Teacher Education, 66*(3), 227–244.

Chapter 1 - Resources

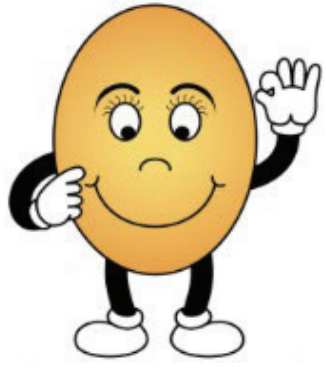
Learning Partners

Rectangle Partner



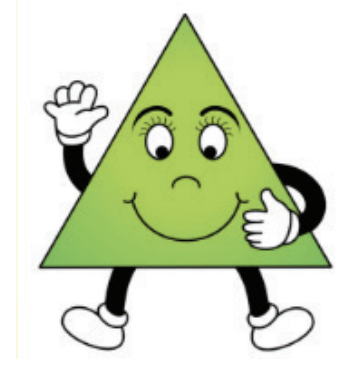
Your partner's name

Oval Partner



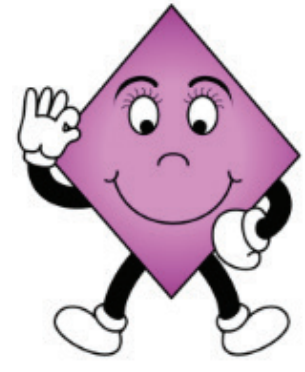
Your partner's name

Triangle Partner



Your partner's name

Diamond Partner



Your partner's name

Learning Partners

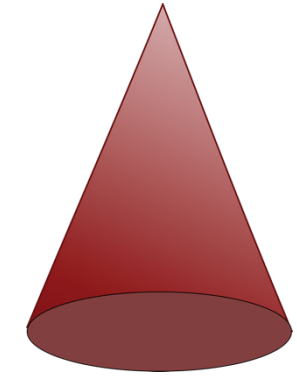
Pyramid Partner

Partner's Name

Cylinder Partner

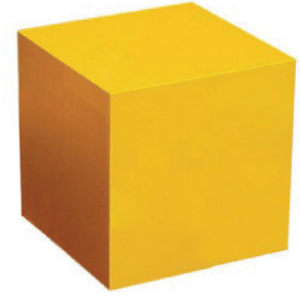
Partner's Name

Cone Partner



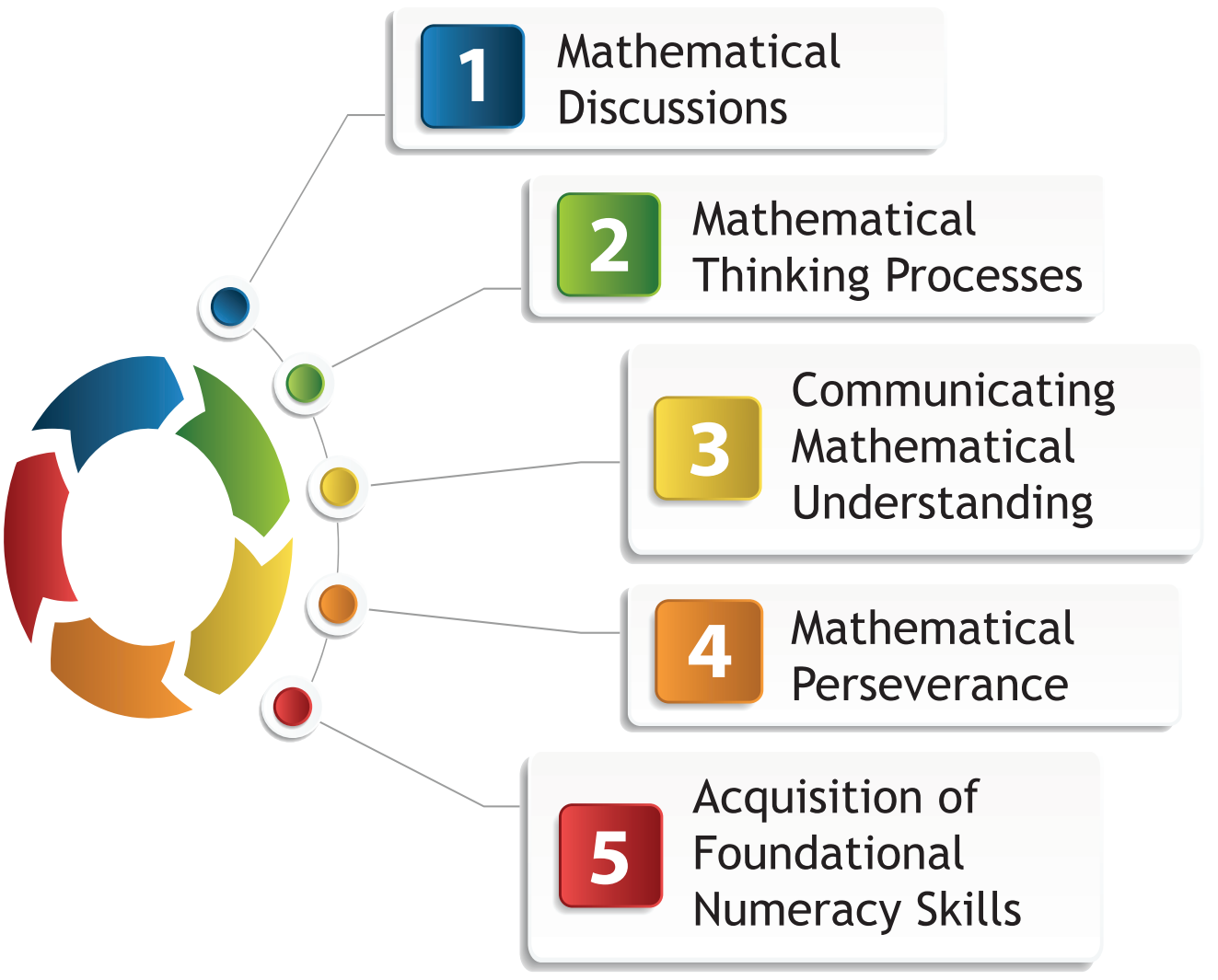
Partner's Name

Cube Partner



Partner's Name

SOAR® High Impact Practices Mathematics TK-2



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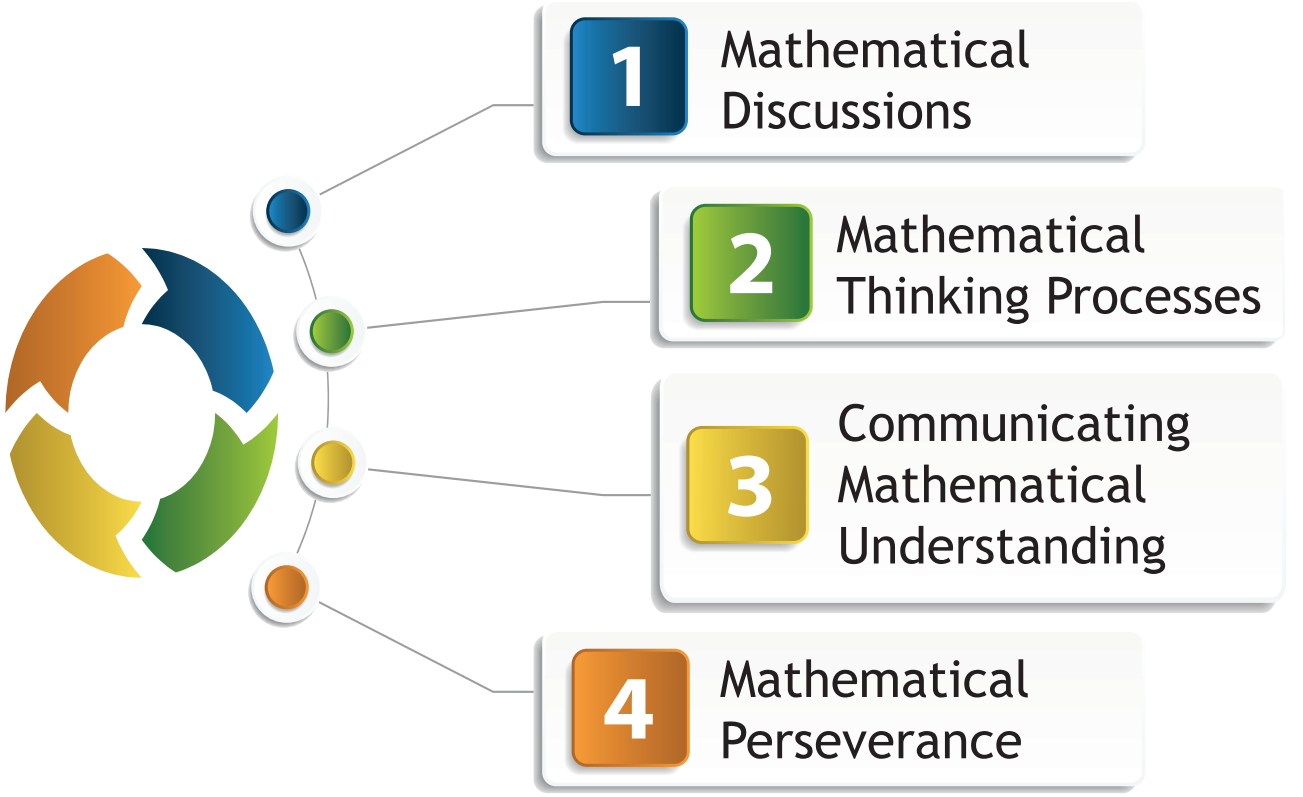


Mathematical Discussions: Math TK-2

HIGH-IMPACT PRACTICE

- Build mathematical conversation skills
- Provide extended, supported, and guided opportunities for students to engage in mathematical discussions

CROSS-CUTTING PRACTICES

Facilitating Acquisition of Academic Language

- Introduce and/or refer to academic language demands of texts and tasks
- Provide extended, guided, and supported opportunities for students to acquire and use the features of academic language

Fostering Metacognition for Mathematical Learning

- Visibly enact metacognitive processes and/or strategies students are expected to use in support of mathematical learning
- Deconstruct metacognitive processes and/or strategies that support mathematical learning

Connecting and Guiding Mathematical Learning

- Elicit student thinking and adjust instruction, supports, and/or mathematical tasks to meet student needs
- Provide written and/or oral feedback during lessons to promote mathematical learning

FOUNDATIONAL PRACTICE

Designing Instruction for Mathematical Thinking and Understanding

- Set mathematical learning targets that are aligned with Math CCSS and the target high-impact practice
- Structure and connect tasks that support the learning targets
- Establish high expectations that support the learning targets and maintain the intellectual rigor of classroom activities and tasks

Mathematical Discussions: Math 3-12

HIGH-IMPACT PRACTICE

- Build mathematical conversation skills
- Provide extended and supported opportunities for students to engage in mathematical discussions

CROSS-CUTTING PRACTICES

Facilitating Acquisition of Academic Language

- Introduce and/or refer to academic language demands of texts and tasks
- Provide extended and supported opportunities for students to acquire and use the features of academic language

Fostering Metacognition for Mathematical Learning

- Visibly enact metacognitive processes and/or strategies students are expected to use in support of mathematical learning
- Deconstruct metacognitive processes and/or strategies that support mathematical learning

Connecting and Guiding Mathematical Learning

- Elicit student thinking and adjust instruction, supports, and mathematical tasks to meet student needs
- Provide written and/or oral feedback during lessons to promote mathematical learning

FOUNDATIONAL PRACTICE

Designing Instruction for Mathematical Thinking and Understanding

- Set mathematical learning targets that are aligned with the Math CCSS and the target high-impact practice
- Structure and connect tasks that support the learning targets
- Establish high expectations that support the learning targets and maintain the intellectual rigor of classroom activities and tasks

Unpacking the Mathematical Discussions Teaching Frame

2

FOCUS QUESTIONS

- What do we mean by Mathematical Discussions, and why is it important?
- What do we mean by each of the Cross-Cutting and Foundational Practices, and why are they important?
- How does the use of videos and vignettes facilitate understanding of each practice?

The CCSS Mathematical Practices

The eight Common Core Mathematical Practices are the foundation for the application of mathematical thinking and understanding by students. In addition, they offer guidance that helps teachers plan and implement instruction to develop students' mathematical thinking and understanding. The practices, described below, are designed to be developed and deepened across the grades starting in kindergarten.

1. ***Make sense of problems and persevere in solving them:*** The first mathematical practice applies to every math problem. It means that students must understand the problem, figure out how to solve it, adjust their approach as needed, and then work until it is finished. Teachers should help students critically analyze problems and plan pathways for solving the problem, instead of jumping right into the solution.
2. ***Reason abstractly and quantitatively:*** When trying to problem-solve, it is important that students understand there are multiple ways to unpack a problem in order to find a solution. Teachers should help students understand the relationships between problem scenarios and mathematical representation, as well as how the symbols represent strategies for a solution.
3. ***Construct viable arguments and critique the reasoning of others:*** This standard is aimed at creating a common mathematical language that can be used to discuss and explain math as well as support or critique others' work. Teachers should provide opportunities for students to participate in extended mathematical discussions to help build student communication skills and guide their ability to create arguments and critique the reasoning of others.
4. ***Model with mathematics:*** Learning to model with mathematics helps students use math appropriately to problem-solve real world situations. This can range from organizing different types of data to using math to help understand real life situations. Teachers should help students see how we can make sense of the world through a mathematical perspective.
5. ***Use appropriate tools strategically:*** Students need to become independent mathematical learners who understand how to select the appropriate tools for solving math problems. Teachers need to help students reflect on their choice of tools for any solution pathway, as well as articulate how, why, and when they should use that specific tool.

6. ***Attend to precision:*** Math involves precision of language, precision of approach, and exact answers. Teachers should foster exactness and attention to detail when students are solving problems, and when they are explaining and justifying their reasoning.
7. ***Look for and make use of structure:*** Identifying similar patterns in mathematics can be used to apply problem-solving approaches to a broad range of math problems. Teachers should help students discern patterns and structures in math.
8. ***Look for and express regularity in repeated reasoning***: In mathematics, it is easy to forget the big picture while working on the details of the problem. Teachers should help students work on applying their mathematical reasoning to various situations and problems.

Teachers should be deeply familiar with the Mathematical Practices, so they can:

- introduce them to students and foster students' ability to track their own mathematical learning and development through the lens of the practices
- plan lessons and tasks with the practices in mind
- design formative assessments of student understanding and implementation of the practices so they can monitor and guide student mathematical thinking and understanding

The **SOAR** Teaching Frames® for Mathematics were developed with the CCSS mathematical standards and mathematical practices in mind. As you go through the manual for Mathematical Discussions, you will see vignettes from elementary classrooms, instructional strategies to guide your implementation of the **SOAR** practices, and lesson ideas to guide use of those strategies. All of these are aligned with the Mathematical Practices. A number of resources are available online to help you introduce the Mathematical Practices to your students, and to guide your teaching so that you foster student development of these practices across all math lessons and activities. We provide links to these in the resource section of this chapter.

High-Impact Practice: *Mathematical Discussions*

What it is

- This practice focuses on structuring, strengthening, and supporting students' ability to engage in student-to-student mathematical discussions.
- These can consist of face-to-face interactions, online dialogues, and student-to-student number talks.

Why it matters

- Learning to converse is an extremely important way to broaden knowledge, enhance understanding, and build community.
- Mathematical discussions involve active processing of information as well as helping to clear up misconceptions and solidify ideas.
- They also make student thinking visible, allowing for more accurate assessment and in-the-moment feedback.

Elements

This practice may be broken down into two important elements. As you review these elements, think about what they might look like when implemented in the classroom.

- Build mathematical conversation skills
- Provide extended and supported opportunities for students to engage in mathematical discussions

Step into the Classroom

Read either the kindergarten or fifth grade vignette that follows. Using the language of the practice and its elements as a guide, underline any evidence that indicates how the teacher addresses Mathematical Discussions. When you have finished, compare what you have underlined to the highlighted evidence you see on pages 31-34 or 35-41.

Kindergarten Vignette

Mr. Ramon has been working with his kindergarten students on having discussions with each other throughout the day. He has established routines and norms where students meet up with different partners to have discussions. For their mathematical discussions, he has worked with his students to ask the question, 'How do you know?', as a way for students to begin to justify their mathematical understanding.

For this lesson, Mr. Ramon has selected the following learning targets that align with the math and ELA CCSS:

K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
MP3 Construct viable arguments and critique the reasoning of others.
MP6 Attend to precision – Students accurately represent each number.
MP7 Look for and make use of structure.
CCSS.ELA-Literacy.SL.K.1: Participate in collaborative conversations with diverse partners about kindergarten topics and texts as well as with peers and adults in small and larger groups.

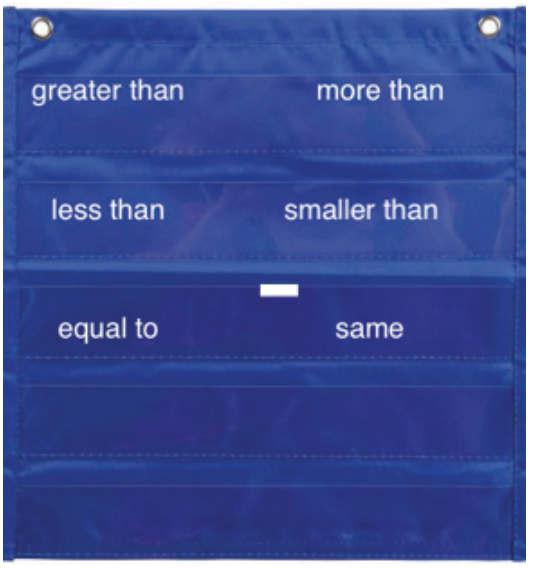
He has also identified language objectives:

Students will be able to orally identify and explain which set is greater than, less than, or equal to using the sentence frames.
Students will be able to explain the structure or strategy they used to justify their answer.

Mr. Ramon begins the math class by having students count down from ten, and skip counting by fives and tens. He reads the math standards that are on the whiteboard to the class.

- I can say which group is greater than by matching or counting the number of objects in both groups.
- I can say which group is less than by matching or counting the number of objects in both groups.
- I can say when groups are equal by matching or counting the number of objects in both groups.
- I can use the words greater than, less than, and equal to when I compare two sets.

He introduces the vocabulary for this lesson. "Today we are going to use special vocabulary to describe our mathematical thinking. (He says the words as he places them in the pocket chart.) 'Greater than.' Say it with me. Greater than. 'Less than.' Say it with me. Less than. 'Equal to.' Say it with me. Equal to. We are going to be comparing groups to see which one is greater than, less than, or equal to."



He asks six students to come to the front of the room. He splits them, four in one group, two in another. He has a student count how many are in each group. He asks the students which group has more students in it. He says, "Yes, Brooklyn's group is greater than Kelsey's group. It has more students in it. 'Greater than' means 'more than'. (He adds 'more than' next to 'greater than' on the pocket chart.) Now, which group is smaller, Brooklyn's or Kelsey's?...Yes, Kelsey's group is less than Brooklyn's group. 'Less than' means 'smaller than'. (He adds 'smaller than' next to 'less than' on the pocket chart.) Let's move people around. Now how many are in Brooklyn's group and how many are in Kelsey's group? How do we know? We know because we counted them. They both have three so we say they are equal to each other. Say that with me. Both groups have three so they are equal to each other. 'Equal' means the 'same'. (He adds that to the pocket chart.) Thank you, boys and girls, you can return to your seats."

Next Mr. Ramon explains, "We are going to play some games today to get better at comparing sets as greater than, less than, or equal to each other. First, we are going to use Legos. Let's count the number of red Legos I have. 1, 2, 3, 4. How do I know I have four red Legos? Turn and tell your triangle partner… Mario, can you tell me what your neighbor said?"
Mario answers, "I know you have four red Legos because we counted them."
"Excellent Mario. You are correct. We counted them. Also, I am very proud of you for stating your response in a complete sentence and using our sentence frame. Now let's count the blue Legos. 1, 2, 3, 4, 5, 6. So we have six blue Legos and four red Legos. How can I compare them? How can I figure out which one is greater than, which one is less than? Maria?"
"We can count them," Maria states.
"Yes, we can. Is there another strategy we can use to figure out if one set is greater than and the other set is less than?"
"You can place them next to each other and see which one is taller," responds Maria.
"Great idea. Let's place them next to one another. Here is the sentence frame we are going to use to compare them. ___ is greater than____. And ___ is less than___. Turn to your triangle partner and discuss if the blue or red Lego tower is greater than or less than using your sentence frame. Ask your partner how you know and respond using the frame, I know because___."

Mr. Ramon moves around the rug to hear students discuss the Legos. He asks some questions and provides feedback to guide the students' discussions.

He has students go back to their seats and distributes Legos to each pair. He provides four more comparison sets to be sure students are understanding the concept. He asks students three questions using their sentence frames to answer. Which set is greater than? Which set is less than? How do you know? On the last example he adds the question, How can we make the sets equal to each other? He has students discuss with their partners using the sentence frame: I can make the set equal to each other by_____.

Mr. Ramon collects the Legos and restates that students were able to know which were greater than and which were less than by counting them and by stacking each Lego tower next to each other. "Now we are going to use our counters to do the same thing we did with the Legos. We are going to figure out which set is greater than, less than, or equal to each other. And we are going to use a strategy to show how we know."

He distributes the colored counters, a die, and a recording sheet. "I am going to model what we are now going to do. I am with my triangle partner. I roll the die. I get a 4. I take four red counters out of the bag and place them on my desk. My partner, Maria, rolls the die and she gets a 3. She takes out three yellow counters. Now we need to answer the questions using our sentence frame. I think the red counters are greater than the yellow counters. Maria, what do you need to ask me?"
"How do you know?" Maria asks.
"I counted," says Mr. Ramon.
Maria says, "Mr. Ramon, is there another strategy we could use to figure it out?"

"We could place the set of yellow counters above the set of red counters and see which is longer. Let's do that. Place your yellow counters above of my red counters. Which is greater? Which is less?" Mr. Ramon asks.

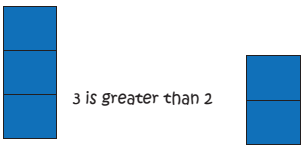
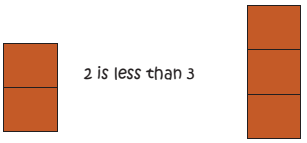
Maria says, "Look, there are more red counters, so the red counters are greater than the yellow counters."

Mr. Ramon says, "And the yellow counters are less than the red counters. Can we make them equal? What would we have to add to the yellow counters to make them equal to the red counters?"

Maria says, "One?"

Mr. Ramon asks, "How do you know?"

"Because if I add one more yellow counter, they will be even. They will be the same," Maria says.

Greater Than • The first set has more than the other set. • The first number is larger than the other number.	
Less Than • The first set has less than the other set. • The first number is smaller than the other.	
Equal to • Both sets or both numbers are the same.	

Mr. Ramon explains that by putting one set above the other, we are able to match them. "You could also stack them like the Legos to see which stack is taller. Or you can play the game take away. I take one of my red counters away and you take a yellow counter away. We keep doing that until one of the stacks is all gone. Then we can ask, Were there more yellow counters or red counters? How do you know?"

"Ok, I'm going to be walking around listening to how you are using less than, greater than, and equal to in your discussions with your partners as you play the game. Remember, if you get stuck you can ask your partner for help or you might ask your partner to clarify what they mean. If your partner gets stuck, you can help guide them and maybe say what you think in a different way. And if you both get stuck, you could look at the chart on your desk, or look at your sentence frames to see if they can help. You could also ask me for help."

Mr. Ramon walks around monitoring students. He stops at one pair and sees they are not discussing but just rolling the die and getting their counters. He asks Kelsey what she could ask Brooklyn to find out if her counters are more or less than Kelsey's. "Brooklyn, are the red counters greater than or less than the yellow counters?"

Mr. Ramon says, "Yes, that is exactly the correct question. So Brooklyn, what is your response?"

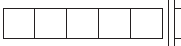
Brooklyn says, "The yellow counters are greater than the red counters. I am going to stack the yellow counters on top of each other and the red counters on top of each other to show that the yellow counters are greater."

"Kelsey, can you show another way that the yellow counters are greater than the red counters?" Mr. Ramon asks.

"We can take one off of each stack to see how many are left. That will also show which one is greater," said Kelsey.
"Brooklyn, is that another way?" Mr. Ramon asks.
"Yes", says Brooklyn. "Let's do that."

He concludes the lesson by asking students to turn to their partners and explain what each of the three terms means. They then complete an assessment where they draw to represent greater than, less than, and equal to.

Name _____Date_____

Draw a tower that is greater than the tower below. _____ is greater than____.	Draw a tower that is less than the tower below. _____ is less than_____.	Draw a tower that is the equal to the tower below. _____ is equal to _____.
		

After underlining any evidence that indicates how the teacher addresses this practice, compare what you have underlined to the highlighted evidence you see on pages 31-34.

Fifth Grade Vignette

CCSS.MATH.CONTENT.5.G.B.3: Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.
CCSS.MATH.CONTENT.5.G.B.4: Classify two-dimensional figures in a hierarchy based on properties.
CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique the reasoning of others.
CCSS.MATH.PRACTICE.MP5 Use appropriate tools strategically.
CCSS.MATH.PRACTICE.MP6 Attend to precision.

CA ELD Standards Part I A 5.1 Contribute to class, group, and partner discussions, including sustained dialogue, by following turn-taking rules, asking relevant questions, affirming others, and adding relevant information.
CA ELD Standards Part I A 5.6 Explain ideas, phenomena, processes, and text relationships (e.g., compare/contrast, cause/effect, problem/solution) based on close reading of a variety of grade-level texts and viewing of multimedia with moderate support.

Ms. Baldwin has developed a series of lessons to teach students mathematical conversation skills and provide extended and supported opportunities for students to engage in mathematical discussions. The focus of the lesson is teaching students mathematical conversation protocols and mathematical conversation skills, specifically the skill of clarify. A protocol the students have been using is Learning Partners. The students have four learning partners, (rhombus, trapezoid, parallelogram, and quadrilateral).

She begins today's lesson by explaining the learning target. "We have been working on using mathematical conversation skills and precise language to explain our thinking to a partner and in discussions. Today we will focus on the conversation rule of 'listen actively' and the conversation skill of 'clarify'." She points to the conversation rules and the conversation skills posters that are on the wall.

Ms. Baldwin projects the standards in student friendly statements and asks a student to read them aloud:
I will:

- use precise language to explain my thinking to a partner.
- explain my mathematical thinking by using models.
- explain how to classify quadrilaterals using properties and attributes.

"We are going to begin by playing the "What is the Term?" game to review what we know about quadrilaterals. Please take out your quadrilaterals learning partners activity sheet. Organize yourselves into groups of four. Each group must have one trapezoid, one rhombus, one parallelogram, and one quadrilateral." As the students get into their small groups the teacher distributes the "What is the Term" decks of cards to each group.

Students sit in their small groups and distribute the cards equally. One side of the card has terms and the other side has definitions. The terms and definitions are quadrilateral, rhombus, kite, parallelogram, rectangle, square, isosceles trapezoid, congruent, perpendicular, angle, parallel, acute, obtuse, and scalene.

Ms. Baldwin tells the class, "I am going to play the game." She selects a card from her deck and reads, "Two lines that are equidistance apart and never intersect. What is the term? All of you read the terms side of your cards to see if you have this term. If you do have the card with this term, confirm that you are correct with your group and then raise your hand." She waits for one member of each group to raise their hand. Once she observes one raised hand from each group, she asks the class. "What is the term?" The class responds, "The term is parallel." "Yes. This activity supports us in learning and using precise mathematical language which is *Math Practice 6* and one of our learning targets for today." Ms. Baldwin points to the Math Practices poster. She continues by telling the class, "We need precise language to more clearly engage in mathematical discussions. Play the game until everyone in your group has used both sides of all their cards."

The students take turns reading their cards, prompting others, and engaging in the game. Ms. Baldwin monitors each group using a clipboard with a protocol she developed to make notes about each student, the conversation skills they are applying, and the language they are using. She listens to make sure all students understand the terms.

When students have finished the task Ms. Baldwin tells them, "Now we are going to classify quadrilaterals based on their characteristics. We are going to use a graphic organizer to classify and discuss quadrilaterals. She distributes the graphic organizer to the class. Ms. Baldwin projects the graphic organizer. "These are the six terms you will use to discuss and classify the quadrilaterals." She reads the terms and definitions aloud to the class.

1. Parallelogram: a simple quadrilateral with two pairs of parallel sides. The opposite sides of a parallelogram are of equal length and the opposite angles of a parallelogram are of equal measure.
2. Quadrilateral: a four-sided figure.
3. Rectangle: a parallelogram with four right angles.
4. Rhombus: a parallelogram with four equal sides.
5. Square: a rectangle with four equal sides.
6. Trapezoid: a quadrilateral with one pair of parallel sides.

"The first column of the graphic organizer has the shapes you will classify. The subsequent columns are one for each of the six terms. I am going to model how to use the graphic organizer by completing the first row. First, I will annotate the shape to identify everything I know about it. These are the rules for highlighting:

- Blue = Parallel sides
- Yellow = Opposite sides are equal
- Orange = Right triangles"

"I am going to think aloud as I work through what I know about this figure. Please listen actively for how I use precise language because that is what I expect you to be able to do. First, I ask myself, 'Does this shape have any right angles?' No. I have not made any highlights. This is important information. Now I go down each column and check if the shape fits the definition. 'Does it fit the definition of parallelogram?" She reads the definition aloud. "No, because it does not have parallel lines. 'Does it fit the definition for a quadrilateral?' Yes, because it has four sides. I place a check mark in that column. 'Does it fit the definition for a rectangle?' No, because it doesn't have four right angles and it is not a parallelogram. 'Does it fit the definition for a rhombus?' No, because it does not have four equal sides and it is not a parallelogram. 'Does it fit the definition for a square?' No, because it does not have four equal sides and it is not a rectangle. 'Does it fit the definition for a trapezoid?' No, because it does not have at least one pair of parallel sides although it is a quadrilateral. This shape is a quadrilateral because it has four sides. It does not have any parallel sides. It does not have any right angles. It does not have any opposite sides that are equal. It is classified as a quadrilateral. Now it is your turn."

Ms. Baldwin continues, "First, you will work independently to highlight each shape using the rules for highlighting. Then you will meet with your rhombus partner. Each of you will explain your annotations to your partner. Once you have explained why and how you highlighted each figure on your graphic organizer, you will work collaboratively to classify the figure. You will use the definitions to question each other about each figure."

Students work independently to highlight each figure on the activity sheet. Ms. Baldwin monitors the students, using her clipboard and protocol. She meets with a few students to provide feedback and ask guiding questions to elicit their mathematical thinking and advance their understanding. She meets with Max because she notices that there is no green on the kite figure. For the kite figure he has highlighted the opposites sides in blue to indicate that they are parallel. He has not highlighted in yellow to indicate that the opposite sides are also equal. She asks, "Max, will you please explain your annotations?

And please use your figure to trace what sides or angles you are talking about." Max responds, "For the kite figure, I highlighted these two lines in blue because they are opposite and parallel. I also highlighted these two lines because they are opposite and parallel." Ms. Baldwin tells Max, "Your use of the mathematical terms and precise language helped me understand your mathematical thinking. Is there anything else your notice about the shape?" Max takes some time to examine the shape. "Yes, I notice that all the sides are equal. I have to highlight them yellow."

Ms. Baldwin pulls the whole class back together. "I notice that all of you have finished highlighting your figures on the activity sheet. Before you meet with your partner, we are going to listen to a model of what it might sound like to have this mathematical discussion." Ms. Baldwin projects the model mathematical conversation. "Listen for the use of mathematical terms and for how the skill of clarify is used to explain the classification. Javier, will you be my partner and read the model with me? You be Student A and I will be Student B." Ms. Baldwin and Javier read the model to the class.

Model

Prompt: What did you highlight and why? How do we classify this shape and why?
Student A: "My second shape looks like I highlighted it all green because I used blue and yellow. I highlighted this pair of lines blue and this pair of lines blue because they are parallel. Then I highlighted each side yellow because the sides are equal. What did you highlight and why?"
Student B: "My shape is also green, but I first highlighted each side yellow because the sides are equal. Then I highlighted the opposite sides blue because they are parallel. How do we classify this shape and why?"
Student A: "Let's start with the first column. Is it a parallelogram?"
Student B: "Yes, because it has two sets of parallel lines. We check that column.
Is it a quadrilateral?"
Student A: "Yes, because it has four sides. We check that column. Is it a rectangle?"
Student B: "Yes, because it is a parallelogram and it has four right angles. We check that column.
Is it a rhombus?"
Student A: "Yes, because it is a parallelogram with four equal sides. We check that column.
Is it a trapezoid?"
Student B: "No, because a trapezoid has only one pair of parallel lines and this shape has two pairs of parallel lines."

"Now meet with your rhombus partner. Each of you explain your highlights and then work collaboratively to classify each figure. Also, use questions to prompt your partner. When we use questions to clarify our partner's thinking, we are using the mathematical conversation skill of clarify. Also remember that a few weeks ago we explored strategies we can use for when a conversation gets 'stuck'. In addition to talking about strategies, we also discussed why, how, and when to use them." Ms. Baldwin points to the Metacognition in Discussions Anchor poster on the wall. "I want you to look to those strategies to help move your conversations forward if the conversation is 'stuck'."

Metacognition in Discussions Anchor Chart

What I can do when I don't understand

- ☐ I can reread the prompt to refocus my thinking.
- ☐ I can summarize my ideas to clarify my thinking.
- ☐ I can ask my partner for help to get back on track.

What I can do when my partner doesn't understand

- ☐ I can paraphrase my partner's ideas to help her refocus her thinking.
- ☐ I can ask a question to prompt my partner to reconsider her thinking.
- ☐ I can explain my thinking to move my partner beyond her misunderstanding.

What we can do when we both need help to move the conversation forward

- ☐ We can clarify why the discussion has stalled to figure out how to move forward.
- ☐ We can retrace the discussion to identify where the breakdown occurred.
- ☐ We can explain why we are stuck and ask for help to enable us to move forward.

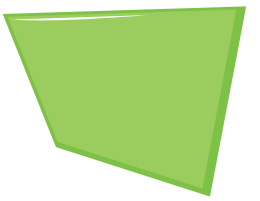
Ms. Baldwin provides students with the following sentence frames to summarize their mathematical discussion and thinking.

A _____ is both a rhombus and a parallelogram because _____.
A _____ is a type of parallelogram.
A _____ is both a rhombus and a rectangle because _____.
A _____ is quadrilateral with exactly one pair of parallel sides.

The students work in pairs and Ms. Baldwin walks around listening to the conversations, asking questions and prompting in order to connect and guide their mathematical discussions, application of conversation skills, and use of appropriate and precise language.

When the discussions have ended Ms. Baldwin pulls everyone back together. "Today we played a game to help us learn and review mathematical terms. This helped us review the precise language we need to explain our mathematical thinking. We have also worked in pairs to discuss and classify quadrilaterals. Now we are going to apply that knowledge to solve a word problem." Ms. Baldwin asks a student to read aloud the word problem that she has projected.

Raquel is making a mosaic using ceramic tiles. She has a tile that is shaped like a quadrilateral. The tile has two right angles and one pair of parallel sides. What type of quadrilateral is the tile piece? Draw a model and support your answer.



"We are going to use the *Share and Compare* protocol and worksheet to capture our thinking and support our mathematical discussions. When you draw your model, you are using MP5 (she points to the Math Practices poster): use appropriate tools strategically. You will draw a model based on the description in order to identify the shape."

SHARE & COMPARE

Name: _____ Partner's Name: _____ Date: _____

☐MP1 – I made sense of the problem and did my best to solve it.	☐ I asked clarifying questions about my partner's idea.
☐MP3 – I explained and justified my thinking using models, numbers, and words.	☐ I answered questions and gave examples to justify my solution.

Problem	
My Solution Pathway	**My Partner's Solution Pathway**
Compare both solution pathways: How is my thinking similar to and/or different from my partner's thinking?	

"Let's read the *Share and Compare* poster to review the steps for the protocol." Students collectively read each step in the protocol. Ms. Baldwin calls on individual students to explain each step and how they will use the *Share and Compare* worksheet.

Isela: "First, we will work independently to write the word problem, draw our model, and support our answer. And be ready to explain our solution pathway to our partner."

Joseph: "Then we listen closely to our partner's solution pathway and write their solution pathway. We also analyze their model. "

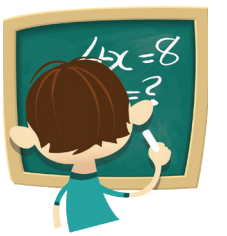
Jamila: "Lastly, we compare our solution pathways. We use the questions to help us with our discussion and then we summarize our thinking. And we are ready to present because you might ask us to fishbowl."

Ms. Baldwin explains, "When you are sharing your solution pathway, these are clarifying questions that you can use to prompt your partner's mathematical thinking, What information did you know about the quadrilateral? How did this information help you determine your answer?"

"For this activity you will work with your trapezoid partner." Students work individually to solve the word problem and then use the Share and Compare worksheet to share and discuss their solution pathways.

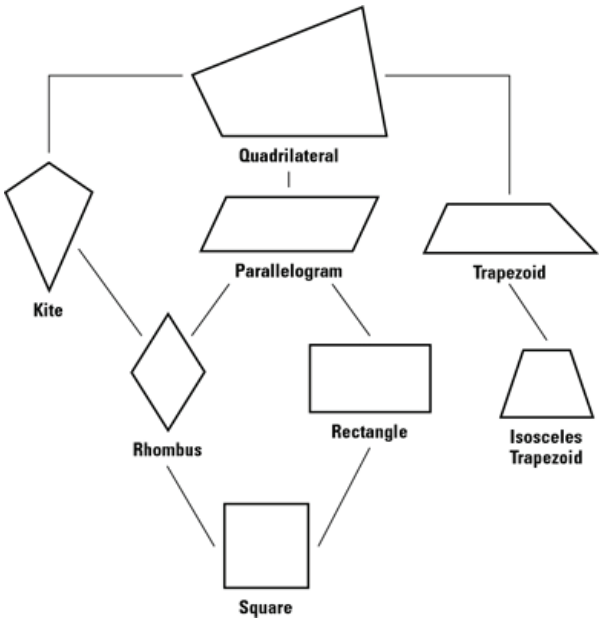
"Let's take our thinking to the next level. What is the term for when we take our thinking to the next level?" Students call out, "Rigor." "Yes. Rigor means working to get more and more precise and thorough in our thinking and knowledge."

Ms. Baldwin projects an image of a quadrilateral family tree. "You will work with a partner to sort a variety of quadrilaterals to develop the hierarchy for quadrilaterals using properties and attributes. This hierarchy is a system in which the shapes are ranked according to the properties and attributes.



You will work with your parallelogram partner to collaboratively explain the quadrilateral family tree. You will begin at the top and work your way down. For each quadrilateral you will explain its properties and attributes.

"Let me model: This is a trapezoid. It is a quadrilateral because it has four sides. It is a trapezoid because it has only two parallel sides. It is ranked below the quadrilateral because it is a type of quadrilateral."



Students work in pairs to explain each step in the hierarchy. Ms. Baldwin has a pair of students present their oral summary of the quadrilateral family tree to the class. She asks students to pose a clarifying question to help their own thinking.

Ms. Baldwin concludes the lesson by projecting the student friendly standards statements. "Today we were working on the mathematical conversation skill of 'clarify' and three learning targets. One way to clarify is to provide examples. Think of one example of what you did to meet each of the learning targets." She gives them think time and then asks them to meet with their quadrilateral partner. The students share their examples with their partners. Ms. Baldwin asks for volunteers to share with the class.

After underlining any evidence that indicates how the teacher addresses this practice, compare what you have underlined to the highlighted evidence you see on pages 35-41.

Cross-Cutting Practice: *Facilitating Acquisition of Academic Language*

What it is

- This practice focuses on structuring, strengthening, and supporting students' acquisition and use of the academic language needed to participate in mathematical tasks.
- Academic language has three features: vocabulary, syntax, and discourse.

Why it matters

- Proficiency in academic language facilitates students' ability to comprehend and analyze text, communicate their mathematical understanding effectively, and acquire mathematical content in all subject areas.
- Academic language development is also associated with student achievement as demonstrated by the correlation between measures of English-language proficiency and content-assessment scores.
- Academic language that is weak or missing is increasingly cited as a major contributor to gaps in achievement between English learners and native speakers of English.

Elements

This practice may be broken down into two important elements. As you review these elements, think about what they might look like in practice in the classroom.

- Introduce and/or refer to the academic language demands of texts and tasks
- Provide extended and supported opportunities for students to acquire and use the features of academic language

Cross-Cutting Practice: *Fostering Metacognition for Mathematical Learning*

What it is

- This practice focuses on the degree to which a teacher visibly enacts and deconstructs metacognitive processes and/or strategies that foster students' metacognitive knowledge.
- Examples of metacognitive processes include self-monitoring, self-assessing, self-questioning, and selecting appropriate strategies.
- Examples of metacognitive strategies include thinking aloud, setting a purpose for learning, and providing a model and nonmodel.

Why it matters

- The use of metacognitive processes and strategies helps students become aware of their own understanding, strengths, and weaknesses.
- These insights enable students to understand what constitutes good work on a given task and what they need to do to complete it successfully.
- This self-knowledge allows students to select, vary, and modify strategies for solving problems and develop independence in directing their own learning, which in turn improves student achievement.

Cross-Cutting Practice: *Connecting and Guiding Mathematical Learning*

What it is

- This practice focuses on how effectively a teacher elicits student thinking and guides the mathematical learning throughout each task and the lesson as a whole, as well as adjusts and supports mathematical tasks to meet the needs of all students in the classroom.
- This practice also includes providing feedback and gradually removing supports to foster students' ability to work flexibly and independently.

Why it matters

- Guiding mathematical learning helps students know exactly what they need to do to meet learning targets.
- It also enables students to clearly express their reasoning and adjust their approaches as needed.
- Specific feedback is fundamental to helping students understand their work and improve on it by raising the level of accountability and engagement with ideas.

Elements

This practice may be broken down into two important elements. As you review these elements, think about what they might look like in practice in the classroom.

- Elicit student thinking and adjust instruction, supports, and mathematical tasks to meet student needs
- Provide written and/or oral feedback during lessons to promote mathematical learning

Foundational Practice: *Designing Instruction for Mathematical Thinking and Understanding*

What it is

- This practice focuses on the design of lessons and learning tasks to promote mathematical learning and support the target High-Impact Practice.
- This practice also focuses on how clearly and directly the teacher aligns mathematical learning targets with the math standards and practices, structures and connects tasks in support of learning targets, and enables students to meet the high expectations of classroom activities.

Why it matters

- Explicit expectations for student learning establish clear goals and enable students to monitor their progress.
- Cohesively designed and logically sequenced instruction, in which lessons work together to support student learning, improves the likelihood that students will achieve the learning targets.
- When teachers have high expectations for students and provide tasks that are engaging and of high interest, students build self-esteem, increase confidence and motivation, and improve academic performance.

Elements

This practice may be broken down into three important elements. As you review these elements, think about what they might look like in practice in the classroom.

- Set mathematical learning targets that are aligned with Math CCSS and the target high-impact practice
- Structure and connect tasks that support the learning target
- Establish high expectations that support the learning targets and maintain the intellectual rigor of classroom activities and tasks.

Conclusion

The purpose of this chapter has been to deepen your understanding of the *Mathematical Discussions Teaching Frame* by unpacking the language of, and rationale for, the instructional practices that comprise it. The classroom vignettes provide additional insight into these practices by illustrating the dynamic and interdependent ways in which they work together in the context of a teaching frame to drive both teacher and student growth. In the next chapter, we will explore instructional strategies that teachers can use to create and implement learning tasks similar to those described in the vignettes.

Chapter 2 - Resources

SOAR
CCSS MATHEMATICAL PRACTICES

STANDARD FOR MATHEMATICAL PRACTICE	STUDENT FRIENDLY LANGUAGE
1. Make sense of problems and perseverence in solving them. 	
2. Reason abstractly and quantitatively. 	
3. Construct viable arguments and critique the reasoning of others. 	
4. Model with mathematics. 	
5. Use appropriate tools strategically. 	
6. Attend to precision. 	
7. Look for and make use of structure. 	
8. Look for and express in repeated reasoning. 	

Kindergarten Vignette

Mr. Ramon has been working with his kindergarten students on having discussions with each other throughout the day. He has established routines and norms where students meet up with different partners to have discussions. For their mathematical discussions, he has worked with his students to ask the question, 'How do you know?', as a way for students to begin to justify their mathematical understanding. MD

For this lesson, Mr. Ramon has selected the following learning targets that align with the math and ELA CCSS:

> K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
> MP3 Construct viable arguments and critique the reasoning of others.
> MP6 Attend to precision – Students accurately represent each number.
> MP7 Look for and make use of structure.
> CCSS.ELA-Literacy.SL.K.1: Participate in collaborative conversations with diverse partners about kindergarten topics and texts as well as with peers and adults in small and larger groups. DI

He has also identified language objectives:

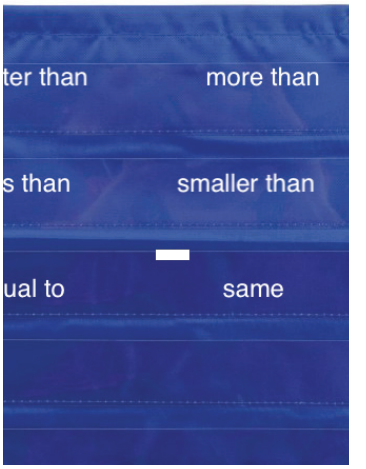
> Students will be able to orally identify and explain which set is greater than, less than, or equal to using the sentence frames.
> Students will be able to explain the structure or strategy they used to justify their answer. DI

Mr. Ramon begins the math class by having students count down from ten, and skip counting by fives and tens. He reads the math standards that are on the whiteboard to the class.

- I can say which group is greater than by matching or counting the number of objects in both groups.
- I can say which group is less than by matching or counting the number of objects in both groups.
- I can say when groups are equal by matching or counting the number of objects in both groups.
- I can use the words greater than, less than, and equal when I compare two sets. DI

He introduces the vocabulary for this lesson. "Today we are going to use special vocabulary to describe our mathematical thinking. (He says the words as he places them in the pocket chart.) 'Greater than.' Say it with me. Greater than. 'Less than.' Say it with me. Less than. 'Equal to.' Say it with me. Equal to. We are going to be comparing groups to see which one is greater than, less than, or equal to." FAAL

He asks six students to come to the front of the room. He splits them, four in one group, two in another. He has a student count how many are in each group. He asks the students which group has more students in it. He says, "Yes, Brooklyn's group is **greater than** Kelsey's group. It has more students in it. 'Greater than' means 'more than.' (He adds 'more than' next to 'greater than' on the pocket chart.) Now, which group is smaller, Brooklyn's or Kelsey's?...Yes, Kelsey's group is **less than** Brooklyn's group. FMML & FAAL 'Less than' means 'smaller than.' (He adds 'smaller than' next to 'less than' on the pocket chart.) FAAL Let's move people around. Now how many are in Brooklyn's group and how many are in Kelsey's group? How do we know? We know because we counted them. They both have three so we say they are **equal** to each other. Say that with me. Both groups have three so they are equal to each other. 'Equal' means the 'same.' (He adds that to the pocket chart.) FMML & FAAL Thank you, boys and girls, you can return to your seats."



Next Mr. Ramon explains, "We are going to play some games today to get better at comparing sets as greater than, less than, or equal to each other. First, we are going to use Legos. Let's count the number of red Legos I have. 1, 2, 3, 4. How do I know I have four red Legos? Turn and tell your triangle partner…Mario, can you tell me what your neighbor said?" Mario answers, "I know you have four red Legos because we counted them." FMML & CGML
"Excellent, Mario. You are correct. We counted them. Also, I am very proud of you for stating your response in a complete sentence and using our sentence frame. CGML Now let's count the blue Legos. 1, 2, 3, 4, 5, 6. So we have six blue Legos and four red Legos. How can I compare them? How can I figure out which one is greater than, which one is less than? Maria?" FMML
"We can count them," Maria states.
"Yes, we can. Is there another strategy we can use to figure out if one set is greater than and the other set is less than?"
"You can place them next to each other and see which one is taller," responds Maria. CGML
"Great idea. Let's place them next to one another. Here is the sentence frame we are going to use to compare them. ___ is greater than____. And ___ is less than___. FAAL Turn to your triangle partner and discuss if the blue or red Lego Tower is greater than or less than using your sentence frame. Ask your partner how do you know and respond using the frame, I know because___." MD

Mr. Ramon moves around the rug to hear students discuss the Legos. He asks some questions and provides feedback to guide the students' discussions. CGML

He has students go back to their seats and distributes Legos to each pair. He provides four more comparison sets to be sure students are understanding the concept. DI He asks students

three questions using their sentence frames to answer. Which set is greater than? Which set is less than? How do you know? On the last example he adds the question, how can we make the sets equal to each other? He has students discuss with their partners using the sentence frame: I can make the set equal to each other by_____. MD & FAAL

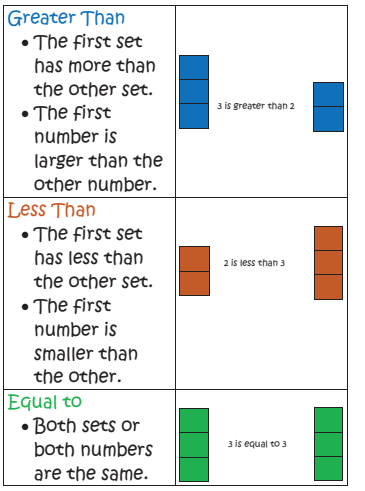
Mr. Ramon collects the Legos and restates that students were able to know which were greater than and which were less than by counting them and by stacking each Lego tower next to each other. "Now we are going to use our counters to do the same thing we did with the Legos. We are going to figure out which set is greater than, less than, or equal to each other. And we are going to use a strategy to show how we know." DI

He distributes the colored counters, a die, and a recording sheet. "I am going to model what we are now going to do. I am with my triangle partner. I roll the die. I get a 4. I take four red counters out of the bag and place them on my desk. My partner, Maria, rolls the die and she gets a 3. She takes out three yellow counters. Now we need to answer the questions using our sentence frame. I think the red counters are greater than the yellow counters. Maria, what do you need to ask me?" FMML
"How do you know?" Maria asks.
"I counted," says Mr. Ramon.
Maria says, "Mr. Ramon, is there another strategy we could use to figure it out?"
"We could place the set of yellow counters above the set of red counters and see which is longer. Let's do that. Place your yellow counters above of my red counters. Which is greater? Which is less?" Mr. Ramon asks.
Maria says, "Look, there are more red counters, so the red counters are greater than the yellow counters."
Mr. Ramon says, "And the yellow counters are less than the red counters. Can we make them equal? What would we have to add to the yellow counters to make them equal to the red counters?"
Maria says, "One?"
Mr. Ramon asks, "How do you know?"
"Because if I add one more yellow counter, they will be even. They will be the same," Maria says. FMML

Mr. Ramon explains that by putting one set above the other, we are able to match them. "You could also stack them like the Legos to see which stack is taller. Or you can play the game take away. I take one of my red counters away and you take a yellow counter away. We keep doing that until one of the stacks is all gone. Then we can ask, Were there more yellow counters or red counters? How do you know?" FMML

"Ok, I'm going to be walking around listening to how you are using less than, greater than, and equal to in your discussions with your partners as you play the game. CGML & FAAL Remember, if you get stuck you can ask your partner for help or you might ask your partner to **clarify** what they mean. If your partner gets stuck, you can help guide them and maybe say what you think

in a different way. And if you both get stuck, you could look at the chart on your desk, or look at your sentence frames to see if they can help. You could also ask me for help." FMML. & MD



Mr. Ramon walks around monitoring students. He stops at one pair and sees they are not discussing but just rolling the die and getting their counters. He asks Kelsey what she could ask Brooklyn to find out if her counters are more or less than Kelsey's. "Brooklyn, are the red counters greater than or less than the yellow counters?" CGML

Mr. Ramon says, "Yes, that is exactly the correct question. So Brooklyn, what is your response?"

Brooklyn says, "The yellow counters are greater than the red counters. I am going to stack the yellow counters on top of each other and the red counters on top of each other to show that the yellow counters are greater."

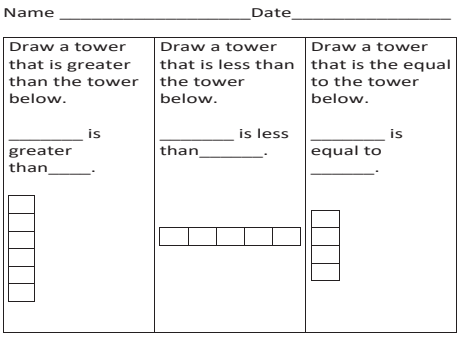
"Kelsey, can you show another way that the yellow counters are greater than the red counters?" Mr. Ramon asks.

"We can take one off of each stack to see how many are left. That will also show which one is greater," said Kelsey.

"Brooklyn, is that another way?" Mr. Ramon asks.

"Yes," says Brooklyn. "Let's do that." CGML

He concludes the lesson by asking students to turn to their partners and explain what each of the three terms means. FAAL They then complete an assessment where they draw to represent greater than, less than, and equal to. FMML


Name _____Date_____

Draw a tower that is greater than the tower below. _____ is greater than_____.	Draw a tower that is less than the tower below. _____ is less than_____.	Draw a tower that is the equal to the tower below. _____ is equal to _____.

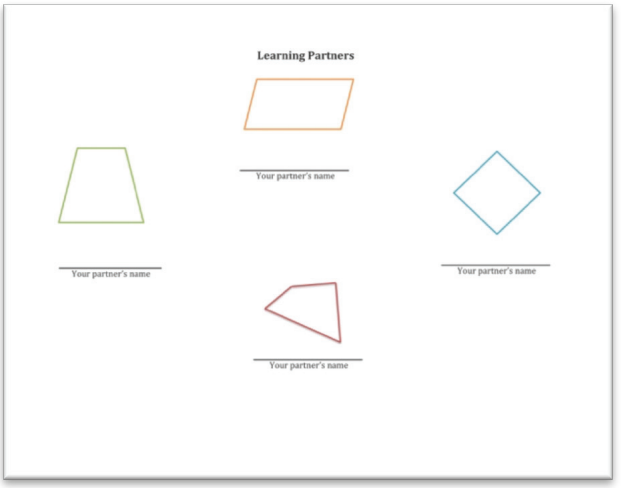
Fifth Grade Vignette

CCSS.MATH.CONTENT.5.G.B.3: Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. DI
CCSS.MATH.CONTENT.5.G.B.4: Classify two-dimensional figures in a hierarchy based on properties. DI

CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique the reasoning of others.
CCSS.MATH.PRACTICE.MP5 Use appropriate tools strategically.
CCSS.MATH.PRACTICE.MP6 Attend to precision. DI

CA ELD Standards Part I A 5.1 Contribute to class, group, and partner discussions, including sustained dialogue, by following turn-taking rules, asking relevant questions, affirming others, and adding relevant information.
CA ELD Standards Part I A 5.6 Explain ideas, phenomena, processes, and text relationships (e.g., compare/contrast, cause/effect, problem/solution) based on close reading of a variety of grade-level texts and viewing of multimedia with moderate support. DI

Ms. Baldwin has developed a series of lessons to teach students mathematical conversation skills and provide extended and supported opportunities for students to engage in mathematical discussions. The focus of the lesson is teaching students mathematical conversation protocols and mathematical conversation skills, specifically the skill of clarify.[MD] A protocol the students have been using is Learning Partners. [MD] The students have four learning partners, (rhombus, trapezoid, parallelogram, and quadrilateral).



She begins today's lesson by explaining the learning target. "We have been working on using mathematical conversation skills and precise language to explain our thinking to a partner and in discussions. Today we will focus on the conversation rule of 'listen actively' and the conversation skill of 'clarify'." [DI] She points to the conversation rules and the conversation skills posters that are on the wall. [MD]

Ms. Baldwin projects the standards in student friendly statements and asks a student to read them aloud:
I will:

- use precise language to explain my thinking to a partner.
- explain my mathematical thinking by using models.
- explain how to classify quadrilaterals using properties and attributes. DI

"We are going to begin by playing the "What is the Term?" game to review what we know about quadrilaterals. Please take out your quadrilaterals learning partners activity sheet. Organize yourselves into groups of four. MD Each group must have one trapezoid, one rhombus, one parallelogram, and one quadrilateral." As the students get into their small groups, the teacher distributes "What is the Term" decks of cards to each group.

Students sit in their small groups and distribute the cards equally. One side of the card has terms and the other side has definitions. The terms and definitions are quadrilateral, rhombus, kite, parallelogram, rectangle, square, isosceles trapezoid, congruent, perpendicular, angle, parallel, acute, obtuse, and scalene. FAAL

Ms. Baldwin tells the class, "I am going to how to play the game." She selects a card from her deck and reads, "'Two lines that are equidistance apart and never intersect.' What is the term? All of you read the terms side of your cards to see if you have this term. If you do have the card with this term, confirm that you are correct with your group and then raise your hand." FAAL She waits for one member of each group to raise their hand. Once she observes one raised hand from each group, she asks the class. "What is the term?" The class responds, "The term is parallel." "Yes. This activity supports us in learning and using precise mathematical language which is *Math Practice 6* and one of our learning targets for today." FAAL Ms. Baldwin points to the Math Practices poster. She continues by telling the class, "We need precise language to more clearly engage in mathematical discussions. FMML Play the game until everyone in your group has used both sides of all their cards."

The students take turns reading their cards, prompting others, and engaging in the game. Ms. Baldwin monitors each group using a clipboard with a protocol she developed to make notes about each student, the conversation skills they are applying, and the language they are using. She listens to make sure all students understand the terms. CGML

When students have finished the task Ms. Baldwin tells them, "Now we are going to classify quadrilaterals based on their characteristics. We are going to use a graphic organizer to classify and discuss quadrilaterals. MD She distributes the graphic organizer to the class. Ms. Baldwin projects the graphic organizer. "These are the six terms you will use to discuss and classify the quadrilaterals." She reads the terms and definitions aloud to the class. FAAL

1. Parallelogram: a simple quadrilateral with two pairs of parallel sides. The opposite sides of a parallelogram are of equal length and the opposite angles of a parallelogram are of equal measure.
2. Quadrilateral: a four-sided figure.
3. Rectangle: a parallelogram with four right angles.
4. Rhombus: a parallelogram with four equal sides.

5. Square: a rectangle with four equal sides.
6. Trapezoid: a quadrilateral with one pair of parallel sides.

"The first column of the graphic organizer has the shapes you will classify. The subsequent columns are one for each of the six terms. I am going to model how to use the graphic organizer by completing the first row. First, I will annotate the shape to identify everything I know about it. FMML These are the rules for highlighting:

- Blue = Parallel sides
- Yellow = Opposite sides are equal
- Orange = Right triangles"

"I am going to think aloud as I work through what I know about this figure. Please listen actively for how I use precise language because that is what I expect you to be able to do. First, I ask myself, 'Does this shape have any right angles?' No, I have not made any highlights. This is important information. Now I go down each column and check if the shape fits the definition. 'Does it fit the definition of parallelogram?''' She reads the definition aloud. "No, because it does not have parallel lines. 'Does it fit the definition for a quadrilateral?' Yes, because it has four sides. I place a check mark in that column. 'Does it fit the definition for a rectangle?' No, because it doesn't have four right angles and it is not a parallelogram. 'Does it fit the definition for a rhombus?' No, because it does not have four equal sides and it is not a parallelogram. 'Does it fit the definition for a square?' No, because it does not have for equal sides and it is not a rectangle. 'Does it fit the definition for a trapezoid?' No, because it does not have at least one pair of parallel sides although it is a quadrilateral. This shape is a quadrilateral because it has four sides. It does not have any parallel sides. It does not have any right angles. It does not have any opposite sides that are equal. It is classified as a quadrilateral. FMML Now it is your turn."

Ms. Baldwin continues, "First, you will work independently to highlight each shape using the rules for highlighting. Then you will meet with your rhombus partner. Each of you will explain your annotations to your partner. Once you have explained why and how you highlighted each figure on your graphic organizer, you will work collaboratively to classify the figure. MD You will use the definitions to question each other about each figure." FAAL

Students work independently to highlight each figure on the activity sheet. Ms. Baldwin monitors the students, using her clipboard and protocol. She meets with a few students to provide feedback and ask guiding questions to elicit their mathematical thinking and advance their understanding. CGML She meets with Max because she notices that there is no green on the kite figure. For the kite figure he has highlighted the opposites sides in blue to indicate that they are parallel. He has not highlighted in yellow to indicate that the opposite sides are also equal. She asks, "Max, will you please explain your annotations? And please use your figure to trace what sides or angles you are talking about." Max responds, "For the kite figure, I highlighted these two lines in blue because they are opposite and parallel. I also highlighted these two lines because they are opposite and parallel." Ms. Baldwin tells Max,

"Your use of the mathematical terms and precise language helped me understand your mathematical thinking. Is there anything else your notice about the shape?" Max takes some time to examine the shape. "Yes, I notice that all the sides are equal. I have to highlight them yellow." CGML

Ms. Baldwin pulls the whole class back together. "I notice that all of you have finished highlighting your figures on the activity sheet. Before you meet with your partner, we are going to listen to a model of what it might sound like to have this mathematical discussion." MD Ms. Baldwin projects the model mathematical conversation. "Listen for the use of mathematical terms and for how the skill of clarify is used to explain the classification. MD Javier, will you be my partner and read the model with me? You be Student A and I will be Student B." Ms. Baldwin and Javier read the model to the class. FMML

Model

Prompt: What did you highlight and why? How do we classify this shape and why?
Student A: "My second shape looks like I highlighted it all green because I used blue and yellow. I highlighted this pair of lines blue and this pair of lines blue because they are parallel. Then I highlighted each side yellow because the sides are equal. What did you highlight and why?"
Student B: "My shape is also green, but I first highlighted each side yellow because the sides are equal. Then I highlighted the opposite sides blue because they are parallel. How do we classify this shape and why?"
Student A: "Let's start with the first column. Is it a parallelogram?"
Student B: "Yes, because it has two sets of parallel lines. We check that column. Is it a quadrilateral?"
Student A: "Yes, because it has four sides. We check that column. Is it a rectangle?"
Student B: "Yes, because it is a parallelogram and it has four right angles. We check that column. Is it a rhombus?"
Student A: "Yes, because it is a parallelogram with four equal sides. We check that column. Is it a trapezoid?"
Student B: "No, because a trapezoid has only one pair of parallel lines and this shape has two pairs of parallel lines." FMML

"Now meet with your rhombus partner. Each of you explain your highlights and then work collaboratively to classify each figure. Also, use questions to prompt your partner. When we use questions to clarify our partner's thinking, we are using the mathematical conversation skill of 'clarify.' MD Also remember that a few weeks

Metacognition in Discussions Anchor Chart

What I can do when I don't understand
- I can reread the prompt to refocus my thinking.
- I can summarize my ideas to clarify my thinking.
- I can ask my partner for help to get back on track.

What I can do when my partner doesn't understand
- I can paraphrase my partner's ideas to help her refocus her thinking.
- I can ask a question to prompt my partner to reconsider her thinking.
- I can explain my thinking to move my partner beyond her misunderstanding.

What we can do when we both need help to move the conversation forward
- We can clarify why the discussion has stalled to figure out how to move forward.
- We can retrace the discussion to identify where the breakdown occurred.
- We can explain why we are stuck and ask for help to enable us to move forward.

ago we explored strategies we can use for when a conversation gets stuck. In addition to talking about strategies, we also discussed why, how, and when to use them." Ms. Baldwin points to the Metacognition in Discussions Chart on the wall. "I want you to look to those strategies to help move your conversations forward if the conversation is stuck." FMML

Ms. Baldwin provides students with the following sentence frames to summarize their mathematical discussion and thinking.

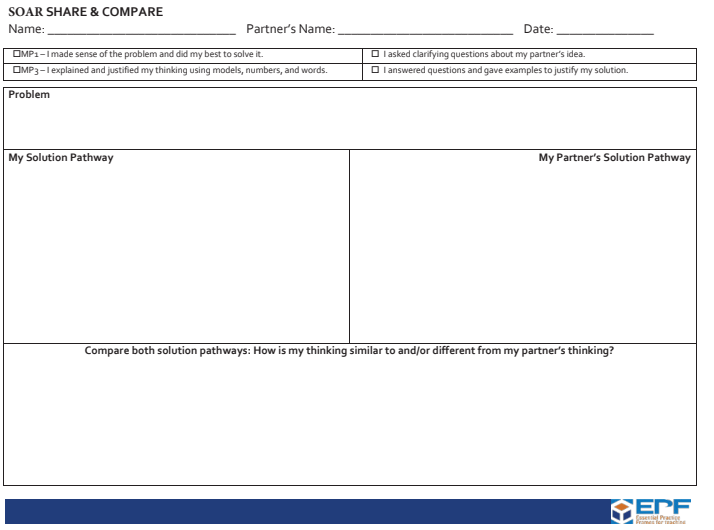
A _____ is both a rhombus and a parallelogram because _____.
A _____ is a type of parallelogram.
A _____ is both a rhombus and a rectangle because _____.
A _____ is quadrilateral with exactly one pair of parallel sides. FAAL

The students work in pairs and Ms. Baldwin walks around listening to the conversations, asking questions and prompting in order to connect and guide their mathematical discussions, application of conversation skills, and use of appropriate and precise language. CGML

When the discussions have ended Ms. Baldwin pulls everyone back together. "Today we played a game to help us learn and review mathematical terms. This helped us review the precise language we need to explain our mathematical thinking. We have also worked in pairs to discuss and classify quadrilaterals. Now we are going to apply that knowledge to solve a word problem." FMML & DI Ms. Baldwin asks a student to read-aloud the word problem that she has projected.

Raquel is making a mosaic using ceramic tiles. She has a tile that is shaped like a quadrilateral. The tile has two right angles and one pair of parallel sides. What type of quadrilateral is the tile piece? Draw a model and support your answer.

We are going to use the *Share and Compare* protocol and worksheet to capture our thinking and support our mathematical discussions. When you draw your model, you are using MP5 (she points to the Math Practices poster): use appropriate tools strategically. You will draw a model based on the description in order to identify the shape." MD



SOAR SHARE & COMPARE

Name: _____ Partner's Name: _____ Date: _____

☐MP1 – I made sense of the problem and did my best to solve it.	☐ I asked clarifying questions about my partner's idea.
☐MP3 – I explained and justified my thinking using models, numbers, and words.	☐ I answered questions and gave examples to justify my solution.

Problem	
My Solution Pathway	**My Partner's Solution Pathway**
Compare both solution pathways: How is my thinking similar to and/or different from my partner's thinking?	

"Let's read the Share and Compare poster to review the steps for the protocol." DI Students collectively read each step in the protocol. Ms. Baldwin calls on individual students to explain each step and how they will use the Share and Compare worksheet. FMML

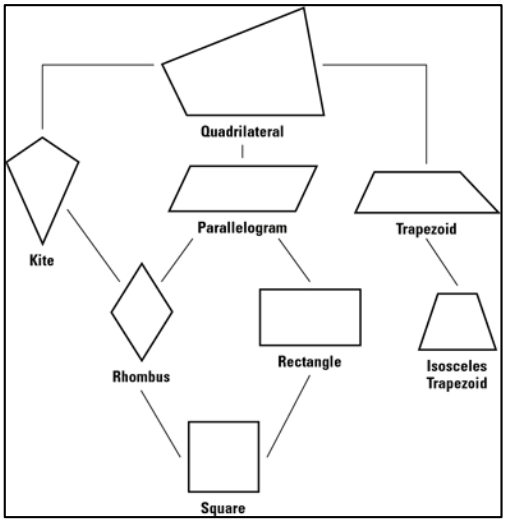
Isela: "First, we will work independently to write the word problem, draw our model, and support our answer. Then we explain our solution pathway to our partner."
Joseph: "Then, we listen closely to our partner's solution pathway and write their solution pathway on our graphic organizer. We also analyze their model."
Jamila: "Lastly, we compare our solution pathways. We use the questions to help us with our discussion and then we summarize our thinking. And we are ready to present because you might ask us to fishbowl."

Ms. Baldwin explains, "When you are sharing your solution pathway, these are clarifying questions that you can use to prompt your partner's mathematical thinking, What information did you know about the quadrilateral? How did this information help you determine your answer?" MD

"For this activity you will work with your trapezoid partner." Students work individually to solve the word problem and then use the Share and Compare worksheet to share and discuss their solution pathways. MD

"Let's take our thinking to the next level. What is the term for when we take our thinking to the next level?" Students call out, "Rigor." "Yes. Rigor means working to get more and more precise and thorough in our thinking and knowledge." FAAL

Ms. Baldwin projects an image of a quadrilateral family tree. "You will work with a partner to sort a variety of quadrilaterals to develop the hierarchy for quadrilaterals using properties and attributes. DI This hierarchy is a system in which the shapes are ranked according to the properties and attributes. FAAL



You will work with your parallelogram partner to collaboratively explain the quadrilateral family tree. MD You will begin at the top and work your way down. For each quadrilateral you will explain its properties and attributes. DI

"Let me model: This is a trapezoid. It is a quadrilateral because it has four sides. It is a trapezoid because it has only two parallel sides. It is ranked below the quadrilateral because it is a type of quadrilateral." FMML

Students work in pairs to explain each step in the hierarchy. MD Ms. Baldwin has a pair of students present their oral summary of the quadrilateral family tree to the class. CGML She asks students to pose a clarifying question to help their own thinking. MD

Ms. Baldwin concludes the lesson by projecting the student friendly standards statements. "Today we were working on the mathematical conversation skill of 'clarify' and three learning targets. One way to clarify is to provide examples. DI Think of one example of what you did to meet each of the learning targets." FMML She gives them think time and then asks them to meet with their quadrilateral partner. The students share their examples with their partners. MD Ms. Baldwin asks for volunteers to share with the class.