



# MATHEMATICAL DISCUSSIONS

## Module Two Manual

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# Mathematical Discussions

<p><b>HIGH-IMPACT PRACTICE</b></p>	<ul style="list-style-type: none"> <li>• Build mathematical conversation skills</li> <li>• Provide extended and supported opportunities for students to engage in mathematical discussions</li> </ul>		
<p><b>CROSS-CUTTING PRACTICES</b></p>	<p><b>Facilitating Acquisition of Academic Language (FAAL)</b></p> <ul style="list-style-type: none"> <li>• Introduce and/or refer to academic language demands of texts and tasks</li> <li>• Provide extended and supported opportunities for students to acquire and use the features of academic language</li> </ul>	<p><b>Fostering Metacognition for Mathematical Learning (FMML)</b></p> <ul style="list-style-type: none"> <li>• Visibly enact metacognitive processes and/or strategies students are expected to use in support of mathematical learning</li> <li>• Deconstruct metacognitive processes and/or strategies that support mathematical learning</li> </ul>	<p><b>Connecting and Guiding Mathematical Learning (CGML)</b></p> <ul style="list-style-type: none"> <li>• Elicit student thinking and adjust instruction, supports, and mathematical tasks to meet student needs</li> <li>• Provide written and/or oral feedback during lessons to promote mathematical learning</li> </ul>
<p><b>FOUNDATIONAL PRACTICE</b></p>	<p><b>Designing Instruction for Mathematical Thinking and Understanding (DI)</b></p> <ul style="list-style-type: none"> <li>• Set mathematical learning targets that are aligned with the Math CCSS and the target high-impact practice</li> <li>• Structure and connect tasks that support the learning targets</li> <li>• Establish high expectations that support the learning targets and maintain the intellectual rigor of classroom activities and tasks</li> </ul>		

# Unpacking the Mathematical Discussions Teaching Frame - Vignette

## Cross Cutting Practice: *Facilitating Acquisition of Academic Language*

### What it is

- This practice focuses on structuring, strengthening, and supporting students' acquisition and use of the academic language needed to participate in mathematical tasks.
- Academic language has three features: vocabulary, syntax, and discourse.

### Why it matters

- Proficiency in academic language facilitates students' ability to comprehend and analyze text, communicate their mathematical understanding effectively, and acquire mathematical content in all subject areas.
- Academic language development is also associated with student achievement as demonstrated by the correlation between measures of English-language proficiency and content-assessment scores.
- Academic language that is weak or missing is increasingly cited as a major contributor to gaps in achievement between English learners and native speakers of English.

### Elements

This practice may be broken down into two important elements. As you review these elements, think about what they might look like when implemented in the classroom.

- Introduce and/or refer to the academic language demands of texts and tasks
- Provide extended and supported opportunities for students to acquire and use the features of academic language

### Step into the Classroom

Read the fifth grade vignette that follows. Using the language of the practice and its elements as a guide, underline any evidence that indicates how the teacher addresses, Facilitating Acquisition of Academic Language.

CCSS.MATH.CONTENT.5.G.B.3: Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.  
CCSS.MATH.CONTENT.5.G.B.4: Classify two-dimensional figures in a hierarchy based on properties.  
CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique the reasoning of others.  
CCSS.MATH.PRACTICE.MP5 Use appropriate tools strategically.  
CCSS.MATH.PRACTICE.MP6 Attend to precision.

Mr. Baldwin has developed a series of lessons to teach students mathematical conversation skills and provide extended and supported opportunities for students to engage in mathematical discussions. The focus of the lesson is teaching students mathematical conversation protocols and mathematical conversation skills, specifically the skill of clarify. A protocol the students have been using is Learning Partners. The students have four learning partners, (rhombus, trapezoid, parallelogram, and quadrilateral).

He begins today's lesson by explaining the learning target. "We have been working on using mathematical conversation skills and precise language to explain our thinking to a partner and in discussions. Today we will focus on the conversation rule of 'listen actively' and the conversation skill of 'clarify'." He points to the conversation rules and the conversation skills posters that are on the wall.

Mr. Baldwin projects the standards in student friendly statements and asks a student to read them aloud:

I will:

- use precise language to explain my thinking to a partner.
- explain my mathematical thinking by using models.
- explain how to classify quadrilaterals using properties and attributes.

"We are going to begin by playing the "What is the Term?" game to review what we know about quadrilaterals. Please take out your quadrilaterals learning partners activity sheet. Organize yourselves into groups of four. Each group must have one trapezoid, one rhombus, one parallelogram, and one quadrilateral." As the students get into their small groups the teacher distributes the "What is the Term" decks of cards to each group.

Students sit in their small groups and distribute the cards equally. One side of the card has terms and the other side has definitions. The terms and definitions are quadrilateral, rhombus, kite, parallelogram, rectangle, square, isosceles trapezoid, congruent, perpendicular, angle, parallel, acute, obtuse, and scalene.

Mr. Baldwin tells the class, "I am going to play the game." He selects a card from her deck and reads, "Two lines that are equidistance apart and never intersect. What is the term? All of you read the terms side of your cards to see if you have this term. If you do have the card with this term, confirm that you are correct with your group and then raise your hand." He waits for one member of each group to raise their hand. Once he observes one raised hand from each group, he asks the class. "What is the term?" The class responds, "The term is parallel." "Yes. This activity supports us in learning and using precise mathematical language which is *Math Practice 6* and one of our learning targets for today." Mr. Baldwin points to the Math Practices poster. He continues by telling the class, "We need precise language to more clearly engage in mathematical discussions. Play the game until everyone in your group has used both sides of all their cards."

The students take turns reading their cards, prompting others, and engaging in the game. Mr. Baldwin monitors each group using a clipboard with a protocol he developed to make notes about each student, the conversation skills they are applying, and the language they are using. He listens to make sure all students understand the terms.

When students have finished the task Mr. Baldwin tells them, "Now we are going to classify quadrilaterals based on their characteristics. We are going to use a graphic organizer to classify and discuss quadrilaterals. He distributes the graphic organizer to the class. Mr. Baldwin projects the graphic organizer. "These are the six terms you will use to discuss and classify the quadrilaterals." He reads the terms and definitions aloud to the class.

1. Parallelogram: a simple quadrilateral with two pairs of parallel sides. The opposite sides of a parallelogram are of equal length and the opposite angles of a parallelogram are of equal measure.
2. Quadrilateral: a four-sided figure.
3. Rectangle: a parallelogram with four right angles.
4. Rhombus: a parallelogram with four equal sides.
5. Square: a rectangle with four equal sides.
6. Trapezoid: a quadrilateral with one pair of parallel sides.

“The first column of the graphic organizer has the shapes you will classify. The subsequent columns are one for each of the six terms. I am going to model how to use the graphic organizer by completing the first row. First, I will annotate the shape to identify everything I know about it. These are the rules for highlighting:

- Blue = Parallel sides
- Yellow = Opposite sides are equal
- Orange = Right triangles”

“I am going to think aloud as I work through what I know about this figure. Please listen actively for how I use precise language because that is what I expect you to be able to do. First, I ask myself, ‘Does this shape have any right angles?’ No. I have not made any highlights. This is important information. Now I go down each column and check if the shape fits the definition. ‘Does it fit the definition of parallelogram?’ He reads the definition aloud. “No, because it does not have parallel lines. ‘Does it fit the definition for a quadrilateral?’ Yes, because it has four sides. I place a check mark in that column. ‘Does it fit the definition for a rectangle?’ No, because it doesn’t have four right angles and it is not a parallelogram. ‘Does it fit the definition for a rhombus?’ No, because it does not have four equal sides and it is not a parallelogram. ‘Does it fit the definition for a square?’ No, because it does not have four equal sides and it is not a rectangle. ‘Does it fit the definition for a trapezoid?’ No, because it does not have at least one pair of parallel sides although it is a quadrilateral. This shape is a quadrilateral because it has four sides. It does not have any parallel sides. It does not have any right angles. It does not have any opposite sides that are equal. It is classified as a quadrilateral. Now it is your turn.”

Mr. Baldwin continues, “First, you will work independently to highlight each shape using the rules for highlighting. Then you will meet with your rhombus partner. Each of you will explain your annotations to your partner. Once you have explained why and how you highlighted each figure on your graphic organizer, you will work collaboratively to classify the figure. You will use the definitions to question each other about each figure.”

Students work independently to highlight each figure on the activity sheet. Mr. Baldwin monitors the students, using her clipboard and protocol. He meets with a few students to provide feedback and ask guiding questions to elicit their mathematical thinking and advance their understanding. He meets with Max because he notices that there is no green on the kite figure. For the kite figure he has highlighted the opposites sides in blue to indicate that they are parallel. He has not highlighted in yellow to indicate that the opposite sides are also equal. He asks, “Max, will you please explain your annotations?”

And please use your figure to trace what sides or angles you are talking about." Max responds, "For the kite figure, I highlighted these two lines in blue because they are opposite and parallel. I also highlighted these two lines because they are opposite and parallel." Mr. Baldwin tells Max, "Your use of the mathematical terms and precise language helped me understand your mathematical thinking. Is there anything else your notice about the shape?" Max takes some time to examine the shape. "Yes, I notice that all the sides are equal. I have to highlight them yellow."

Mr. Baldwin pulls the whole class back together. "I notice that all of you have finished highlighting your figures on the activity sheet. Before you meet with your partner, we are going to listen to a model of what it might sound like to have this mathematical discussion." Mr. Baldwin projects the model mathematical conversation. "Listen for the use of mathematical terms and for how the skill of clarify is used to explain the classification. Javier, will you be my partner and read the model with me? You be Student A and I will be Student B." Mr. Baldwin and Javier read the model to the class.

### Model

Prompt: What did you highlight and why? How do we classify this shape and why?

Student A: "My second shape looks like I highlighted it all green because I used blue and yellow. I highlighted this pair of lines blue and this pair of lines blue because they are parallel. Then I highlighted each side yellow because the sides are equal. What did you highlight and why?"

Student B: "My shape is also green, but I first highlighted each side yellow because the sides are equal. Then I highlighted the opposite sides blue because they are parallel. How do we classify this shape and why?"

Student A: "Let's start with the first column. Is it a parallelogram?"

Student B: "Yes, because it has two sets of parallel lines. We check that column.

Is it a quadrilateral?"

Student A: "Yes, because it has four sides. We check that column. Is it a rectangle?"

Student B: "Yes, because it is a parallelogram and it has four right angles. We check that column.

Is it a rhombus?"

Student A: "Yes, because it is a parallelogram with four equal sides. We check that column.

Is it a trapezoid?"

Student B: "No, because a trapezoid has only one pair of parallel lines and this shape has two pairs of parallel lines."



“Now meet with your rhombus partner. Each of you explain your highlights and then work collaboratively to classify each figure. Also, use questions to prompt your partner. When we use questions to clarify our partner’s thinking, we are using the mathematical conversation skill of clarify. Also remember that a few weeks ago we explored strategies we can use for when a conversation gets ‘stuck’. In addition to talking about strategies, we also discussed why, how, and when to use them.” Mr. Baldwin points to the Metacognition in Discussions Anchor poster on the wall. “I want you to look to those strategies to help move your conversations forward if the conversation is ‘stuck’.”

Metacognition in Discussions Anchor Chart

*What I can do when I don't understand*

- I can reread the prompt to refocus my thinking.
- I can summarize my ideas to clarify my thinking.
- I can ask my partner for help to get back on track.

*What I can do when my partner doesn't understand*

- I can paraphrase my partner’s ideas to help her refocus her thinking.
- I can ask a question to prompt my partner to reconsider her thinking.
- I can explain my thinking to move my partner beyond her misunderstanding.

*What we can do when we both need help to move the conversation forward*

- We can clarify why the discussion has stalled to figure out how to move forward.
- We can retrace the discussion to identify where the breakdown occurred.
- We can explain why we are stuck and ask for help to enable us to move forward.

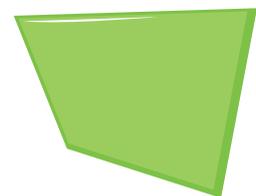
Mr. Baldwin provides students with the following sentence frames to summarize their mathematical discussion and thinking.

- A \_\_\_\_\_ is both a rhombus and a parallelogram because \_\_\_\_\_.
- A \_\_\_\_\_ is a type of parallelogram.
- A \_\_\_\_\_ is both a rhombus and a rectangle because \_\_\_\_\_.
- A \_\_\_\_\_ is quadrilateral with exactly one pair of parallel sides.

The students work in pairs and Mr. Baldwin walks around listening to the conversations, asking questions and prompting in order to connect and guide their mathematical discussions, application of conversation skills, and use of appropriate and precise language.

When the discussions have ended Mr. Baldwin pulls everyone back together. “Today we played a game to help us learn and review mathematical terms. This helped us review the precise language we need to explain our mathematical thinking. We have also worked in pairs to discuss and classify quadrilaterals. Now we are going to apply that knowledge to solve a word problem.” Mr. Baldwin asks a student to read aloud the word problem that he has projected.

*Raquel is making a mosaic using ceramic tiles. He has a tile that is shaped like a quadrilateral. The tile has two right angles and one pair of parallel sides. What type of quadrilateral is the tile piece? Draw a model and support your answer.*



"We are going to use the *Share and Compare* protocol and worksheet to capture our thinking and support our mathematical discussions. When you draw your model, you are using

MP5 (he points to the Math Practices poster): use appropriate tools strategically. You will draw a model based on the description in order to identify the shape."

"Let's read the *Share and Compare* poster to review the steps for the protocol." Students collectively read each step in the protocol. Mr. Baldwin calls on individual students to explain each step and how they will use the *Share and Compare* worksheet.

Isela: "First, we will work independently to write the word problem, draw our model, and support our answer. And be ready to explain our solution pathway to our partner."

Joseph: "Then we listen closely to our partner's solution pathway and write their solution pathway. We also analyze their model. "

Jamila: "Lastly, we compare our solution pathways. We use the questions to help us with our discussion and then we summarize our thinking. And we are ready to present because you might ask us to fishbowl."

Mr. Baldwin explains, "When you are sharing your solution pathway, these are clarifying questions that you can use to prompt your partner's mathematical thinking, What information did you know about the quadrilateral? How did this information help you determine your answer?"

"For this activity you will work with your trapezoid partner." Students work individually to solve the word problem and then use the *Share and Compare* worksheet to share and discuss their solution pathways.

"Let's take our thinking to the next level. What is the term for when we take our thinking to the next level?" Students call out, "Rigor." "Yes. Rigor means working to get more and more precise and thorough in our thinking and knowledge."

Mr. Baldwin projects an image of a quadrilateral family tree. "You will work with a partner to sort a variety of quadrilaterals to develop the hierarchy for quadrilaterals using properties and attributes. This hierarchy is a system in which the shapes are ranked according to the properties and attributes.

SHARE & COMPARE

Name: \_\_\_\_\_ Partner's Name: \_\_\_\_\_ Date: \_\_\_\_\_

<input type="checkbox"/> MP1 – I made sense of the problem and did my best to solve it.	<input type="checkbox"/> I asked clarifying questions about my partner's idea.
<input type="checkbox"/> MP3 – I explained and justified my thinking using models, numbers, and words.	<input type="checkbox"/> I answered questions and gave examples to justify my solution.

Problem	
My Solution Pathway	My Partner's Solution Pathway
Compare both solution pathways: How is my thinking similar to and/or different from my partner's thinking?	



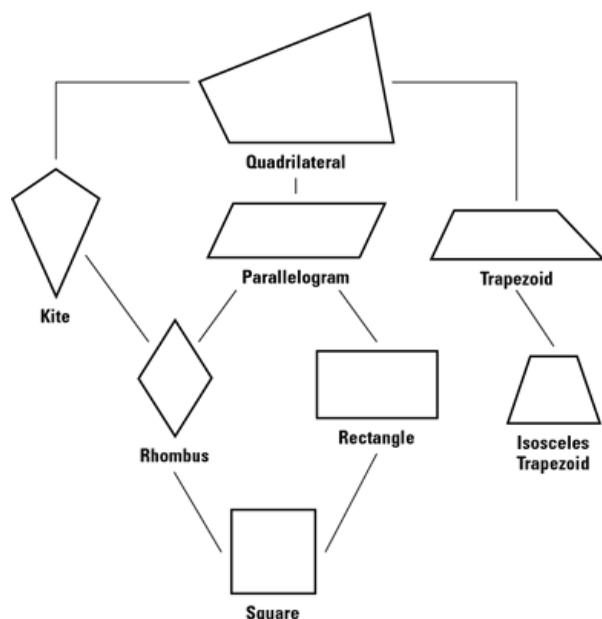
You will work with your parallelogram partner to collaboratively explain the quadrilateral family tree. You will begin at the top and work your way down. For each quadrilateral you will explain its properties and attributes.

“Let me model: This is a trapezoid. It is a quadrilateral because it has four sides. It is a trapezoid because it has only two parallel sides. It is ranked below the quadrilateral because it is a type of quadrilateral.”

Students work in pairs to explain each step in the hierarchy. Mr. Baldwin has a pair of students present their oral summary of the quadrilateral family tree to the class. He asks students to pose a clarifying question to help their own thinking.

Mr. Baldwin concludes the lesson by projecting the student friendly standards statements. “Today we were working on the mathematical conversation skill of ‘clarify’ and three learning targets. One way to clarify is to provide examples. Think of one example of what you did to meet each of the learning targets.” He gives them think time and then asks them to meet with their quadrilateral partner. The students share their examples with their partners. Mr. Baldwin asks for volunteers to share with the class.

After underlining any evidence that indicates how the teacher addresses this practice, compare what you have underlined to the highlighted evidence you see on pages 35-41.



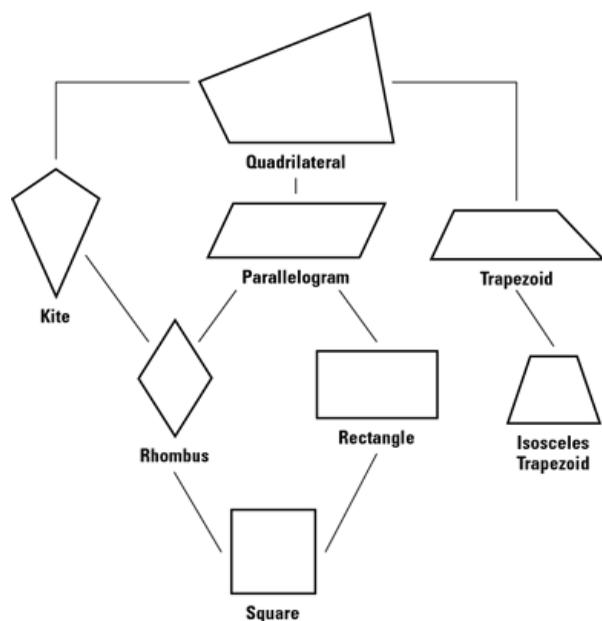
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# Instructional Strategies that Support the Implementation of Mathematical Discussions

## FOCUS QUESTIONS

- What are some examples of instructional strategies teachers can use to implement the teaching practices in the *Mathematical Discussions Teaching Frame*?
- How are these strategies similar to or different from ones you currently use in your teaching?
- What is one way you could use one of these strategies in your classroom?

## Instructional Strategy—*Share and Compare*

The *Share and Compare* strategy is designed for students to practice explaining and justifying their approach to solving a problem with a partner. Students work on their own to solve a problem and model their thinking using manipulatives or drawings, then take turns explaining their thinking to their partner. Next, students document their partner's thinking. Finally, students work collaboratively to compare their approaches, documenting similarities and differences.

**Why Use This Strategy:** The *Share and Compare* strategy is a way to provide students with structured opportunities to communicate their mathematical reasoning during problem solving. Students are provided with language supports to build a logical set of statements to explain and justify their solution pathway; they use manipulatives, drawings, numbers, and words to represent or document their arguments and refer to these models as they take turns communicating their thinking with a partner. Furthermore, partners have the opportunity to make connections between their solutions, which helps students develop deeper conceptual understanding and links to big ideas in mathematics. This strategy specifically targets the first part of MP3 – Construct viable arguments and critique the reasoning of others. It helps diverse learners (struggling readers, ELs, SWDs, etc.) to acquire the language skills associated with explaining and justifying their mathematical thinking. Students also engage in MP4 – Model with mathematics as they use manipulatives, equations, and drawings to show their mathematical thinking about the problem. Additionally, students who are able to go back and forth between decontextualizing and contextualizing the problem as they are explaining their solution, hone their ability to reason abstractly, which is MP2—Reason abstractly and quantitatively. The graphic organizer, strategic questions, prompt and response starters, and socio-cultural learning structure of the strategy provide ELs with appropriate supports and practice to engage in various MPs and communicate their ideas in the mathematics classroom.

**When to Use This Strategy:** This strategy is meant to be used after students have unpacked a problem or task. They should have an opportunity to organize their thinking and construct their solution pathway with concrete models using manipulatives, drawings, numbers, and words prior to engaging in the strategy. Once students have had ample time to construct an argument, even if they have not arrived at a final solution, you can use the *Share and Compare* strategy to provide students with a structured opportunity to document and communicate their thinking to a partner.

**How to Use This Strategy:** Solve the problem or task you have selected in as many ways as you can before you present it to the students, so that you may anticipate students' solution pathways. Provide students with enough time to unpack or make sense of the problem or task and tools (e.g., counters, graph paper, ten frames) to begin solving. Once most students have solved (not all students have to finish), use *Share and Compare* to guide students as they construct an argument for their solution pathway. Provide prompt and response starters and the graphic organizer to support students' thinking and conversations. The lesson should culminate with two or three student pairs (selected by the teacher) presenting their *Share and Compare* conversations/writing for the class.

<b>Teacher Facilitation</b>	
<b>Before <i>Share and Compare</i></b>	<ul style="list-style-type: none"> <li>• Students make sense of the problem or task</li> <li>• They are allowed ample time and tools to work on solving the problem/completing the task</li> </ul>
<b><i>Share and Compare</i> Strategy</b>	<ul style="list-style-type: none"> <li>• Teacher hands out the graphic organizer, prompt and response starters, and uses the poster to introduce/review the purpose of the strategy</li> <li>• Each student works on creating a mathematical model to demonstrate their solution pathway; students may use manipulatives, drawings, numbers, and words; teacher monitors and supports individual students as needed</li> <li>• Students document their thinking using the graphic organizer; teacher monitors and supports individual students as needed</li> <li>• Students are paired up</li> <li>• Teacher refers to poster and reviews prompt and response starters as needed</li> <li>• Students take turns explaining and justifying their solutions; teacher monitors partner discussions and provides feedback as needed</li> <li>• Teacher prompts students to document their partner's thinking; teacher monitors and supports individual students as needed</li> <li>• Students use the graphic organizer and prompt and response starters to compare their solutions orally and then writing; teacher monitors partner discussions and provides feedback as needed</li> <li>• Teacher selects two or three student pairs to present their <i>Share and Compare</i> conversations/writing to the class</li> </ul>

## Supporting Materials for Share and Compare Prompt and Response Starters

SOAR Share and Compare Prompt & Response Starters

	Prompt Starters	Response Starters
<b>Explain &amp; Justify</b>	<ul style="list-style-type: none"> <li>• How did you approach the problem?</li> <li>• What did you do first?</li> <li>• Why did you...?</li> <li>• Can you explain how...</li> <li>• In your solution you...Why?</li> <li>• What does ... represent?</li> <li>• Tell me more about...</li> </ul>	<ul style="list-style-type: none"> <li>• First, I...</li> <li>• Next, I...</li> <li>• Then, I...</li> <li>• I used ... to show...</li> <li>• The reason I ... is because...</li> <li>• The ... represents...</li> <li>• I would like to add...</li> </ul>
<b>Compare</b>	<ul style="list-style-type: none"> <li>• How are our solutions alike and different?</li> <li>• How would you describe each solution?</li> <li>• How is my thinking similar to yours?</li> <li>• How is my thinking different from yours?</li> <li>• What is one connection you can make?</li> </ul>	<ul style="list-style-type: none"> <li>• Our solutions are alike because...</li> <li>• Our solutions are different because...</li> <li>• In my solution...</li> <li>• In your solution...</li> <li>• Our thinking is similar because...</li> <li>• Our thinking is different because...</li> <li>• One connection is...</li> </ul>



### SOAR SHARE & COMPARE

Name: \_\_\_\_\_ Partner's Name: \_\_\_\_\_ Date: \_\_\_\_\_

<input type="checkbox"/> MP1 – I made sense of the problem and did my best to solve it.	<input type="checkbox"/> I asked clarifying questions about my partner's idea.
<input type="checkbox"/> MP3 – I explained and justified my thinking using models, numbers, and words.	<input type="checkbox"/> I answered questions and gave examples to justify my solution.
<b>Problem</b>	
<b>My Solution Pathway</b>	<b>My Partner's Solution Pathway</b>
Compare both solution pathways: How is my thinking similar to and/or different from my partner's thinking?	

**Lesson Example:** Grade 4, Number and Operations--Fractions

(4.NFc) Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

*Isabel wants to have a barbecue to celebrate her dad's birthday. She needs to buy  $\frac{2}{5}$  pound of meat per person. The packages are sold in packs of 1 pound. If 11 people attend, how many pounds of meat does Isabel need? How many packages will she need to buy? Isabel is also hoping to have leftover meat for her lunch tomorrow. Will she have enough? Use a model and an equation to show your thinking. Use words to explain and justify the steps in your solution.*

- Teacher hands out the graphic organizer, prompt and response starters, and uses the poster to introduce/review the purpose of the strategy.
  - o Teacher: You've all had an opportunity to make sense of the problem and solve it. I know some of you aren't finished with your solutions and that's ok. You all have some ideas about how to solve the problem and we will now *Share and Compare* (points to poster) our thinking with a partner. Why should mathematicians share and compare their thinking?
  - o Alicia: Mathematicians learn from each other. Sometimes if you're stuck, listening to another person's way of solving can help you understand.
  - o Mario: I agree with Alicia. Listening to another mathematician's ideas is helpful, but sharing your own thinking out loud helps, too.
  - o Teacher: I see a lot of you agree. Who would like to add to what Mario said?
  - o Clarissa: When I get stuck, explaining my thinking to a partner helps me because I have to really understand the math to explain it to my partner. Like when my partner asks questions and I have to be clear when I explain, it helps me see my mistakes.
  - o Teacher: Yes (points to poster) "Today we do it together so that tomorrow I can do it on my own!" and engaging in math discussion to share our thinking helps us understand the math better, which is why we use this strategy. What will help us have a successful math discussion today?
  - o Ruben: We need to follow our conversation norms (points to norms poster) and use our prompt and response starters if we need them.
  - o Teacher: Good. Today I want you to focus on listening actively to your partner and asking questions to clarify ideas. What might you ask someone to clarify?
  - o Amy: I might ask, "How did you get that?"
  - o Teacher: How could you answer?
  - o Beatriz: I would point to my model and say, "First I..." and keep explaining my steps.
- Each student works on creating a mathematical model to demonstrate their solution pathway; students may use manipulatives, drawings, numbers, and words. Teacher monitors and supports individual students as needed.
  - o Teacher: Now, use the next 10 minutes to work on showing your thinking on the graphic organizer. You may use models, drawings, equations, and words. I see some of you are also using fraction strips and others are using number lines. Just make sure you take this time to document your solution somehow. You should refer to it as you explain just as Beatriz said.

- o Gloria: I want to use my fraction strips, but I'm not sure how.
- o Teacher: Well, what do you want to show?
- o Gloria: I want to show the meat for each person. Like  $\frac{2}{5}$  for one person and  $\frac{2}{5}$  for the next person until I show the meat for 11 people.
- o Teacher: OK. So, if you're dealing with fifths your denominator is 5, and what does that mean?
- o Gloria: That each pound is being divided into 5 equal parts.
- o Teacher: Yes. And how would you show one of those parts?
- o Gloria: With  $\frac{1}{5}$ ? (Finds  $\frac{1}{5}$  fraction strip and places it on the table.) So, I need one more fifth (places another  $\frac{1}{5}$  fraction strip next to the other one) to show the meat for one person. And if I do it again (places 2 more fifths on the table) now I have the meat for two people.
- o Teacher: That makes sense to me. How much meat is that?
- o Gloria: It's  $\frac{4}{5}$  pound. But, now I only have one more piece—one more fifth. How will I show the other 10 people?
- o Teacher: Well, maybe you can borrow someone else's pieces or maybe you can draw it. What I find more interesting, Gloria, is why you only have one more fifth left. What does that mean?
- o Gloria: I'm not sure.
- o Teacher: What would happen if you add it to what you have?
- o Gloria: Then it would be a whole.
- o Teacher: A whole what?
- o Gloria: A whole pound of meat...one pound of meat. So, for two people it's almost 1 pound, but not exactly. So that means that  $\frac{2}{5}$  is less than  $\frac{1}{2}$ , but it's close.
- o Teacher: Hmm. Estimating helps us to get close to the amount and it can be a way to check if our answer is reasonable. Good thinking, Gloria.
- Students document their thinking using the graphic organizer. Teacher monitors and supports individual students as needed.
  - o Teacher: Steven, I see that you've drawn a model to show your thinking. How might you represent that model with an equation?
  - o Steven: Add it. Like add each one to get the total?
  - o Teacher: Try it and see if it makes sense when you explain it to your partner.
- Students are paired up.
  - o Teacher: Ok. Now that you've organized and documented your solutions on the graphic organizer, I want you to find your partner. You will be working with your "perpendicular lines" partner today.
- Teacher refers to poster and reviews prompt and response starters as needed.
  - o Remember to take turns explaining your solutions (points to poster). You may use your prompt and response starters to help you.
- Students take turns explaining and justifying their solutions.
  - o Mia: How did you solve the problem? What did you do first?
  - o Nelson: First, I used my fraction strips. I used  $\frac{1}{5}$  four times and that is the amount of meat for 2 people.

- o Teacher: Mia, I can see from your body language that you need Nelson to clarify. What can you say to him?
- o Mia: (Looks at prompt and response starters). Can you explain how you got that?
- o Nelson: Yes. So, (lays out two  $\frac{1}{5}$  fraction pieces) these make  $\frac{2}{5}$  and that's the meat for one person (lays out two more  $\frac{1}{5}$  pieces) and these make  $\frac{2}{5}$  and that's the meat for one more person. Together, that's  $\frac{4}{5}$  and that's the meat for two people.
- o Mia: Ok. So what you are saying is that for every two people you can use  $\frac{4}{5}$ . What did you do next?
- o Nelson: I only had one more  $\frac{1}{5}$  strip, so that means it's almost a whole pound. So, for every two people that's almost a pound of meat you need.
- o Mia: I heard you say that  $\frac{4}{5}$  pound is close to one whole pound. How will that help you solve the problem?
- o Nelson: I can get close (points to graphic organizer). So, for four people it's 2 pounds and for six people it's 3 pounds, for 8 people it's 4 pounds, for 10 people it's 5 pounds. I think the answer is close to 5 pounds.
- o Mia: Why does that make sense? The problem says 11 people, not 10 people.
- o Nelson: Yes, but for every two people there is an extra  $\frac{1}{5}$  of meat and that's enough extra for the last person with enough left over for lunch.
- o Mia: Your solution is more like an estimate. How can you find the exact amount?
- o Nelson: That's where I got stuck, but now I see that I can just write an equation. I think I can add it all up. How did you solve the problem, Mia?
- o Mia: I used a number line (points to graphic organizer) to show...
- Teacher prompts students to document their partner's thinking. Teacher monitors students as they work and supports individual students as needed.
  - o Teacher: You've all had an opportunity to share your ideas and listen to someone else's ideas. Think about your partner's ideas. How did they solve the problem? Try to explain or show your partner's thinking on the graphic organizer. Get ready to work on comparing both solutions.
  - o Teacher: Olivia, I see that you are drawing a number line to show your thinking. How will you attend to precision like a mathematician?
  - o Olivia: I can use a ruler to make sure I divide each section into 5 equal parts.
  - o Teacher: Mathematician Olivia just shared how she will be using a ruler to help her draw her fraction number line precisely. Mathematicians attend to precision and use tools to help them. Think about the tools you are using and how you are documenting your thinking with precision.
- Students use the graphic organizer and prompt and response starters to compare their solutions orally and then writing. Teacher monitors partner discussions and provides feedback as needed.
  - o Teacher: Now I want you to collaborate with your partner to discuss how your solutions are similar and different. You may use your prompt and response starters to help you with your discussion. When you're finished discussing, work with your partner to write a paragraph to compare your solutions.

- o Nelson: In my solution I used repeated addition to show how  $4/5 + 4/5 + 4/5 + 4/5 + 4/5 + 2/5 = 22/5$  pounds of meat or 4 and  $2/5$  pounds of meat. In your solution you also used repeated addition, but you added  $2/5$  eleven times.
- o Mia: In my solution I used a number line to model my thinking and I shaded  $2/5$  eleven times until I got to 4 and  $2/5$  on the number line. In your solution you used fraction strips to model your thinking, but your model showed that  $4/5$  was enough meat for two people and that helped you estimate that you need about 1 pound of meat per person.
- o Teacher: I see that both of you used repeated addition. How could you show your thinking using multiplication?
- o Mia: Oh, I added  $2/5$  eleven times, so that would be  $11 \times 2/5$ .
- o Teacher: Hmm. What might that equation look like, Mia? What would yours look like, Nelson?
- o Nelson: It would be  $5 \times 4/5 + 2/5 = 22/5$  and Mia's would be  $11 \times 2/5 = 22/5$ .
- o Mia: How are both equations similar and different?
- o Nelson: We both have the same answer but multiplied in different ways.
- o Teacher: Excellent, Mia and Nelson. I like the way you're working together and using the prompt and response starters to compare your solutions.
- Teacher selects two or three student pairs to present their *Share and Compare* conversations/ writing to the class. "I heard many of you using your prompt and response starters to help you compare your solutions. I also saw that mathematicians were referring to the details of their solutions, such as your models and equations. Most of you were able to finish writing your paragraphs. Let's listen to a few examples."

## Instructional Strategy- Resources





# SOAR

## SHARE & COMPARE

Today we do it together so that tomorrow I can do it on my own.

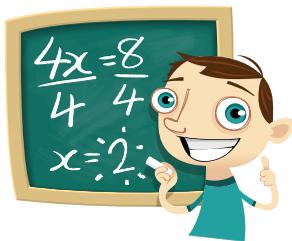
I work on the problem and show my thinking.



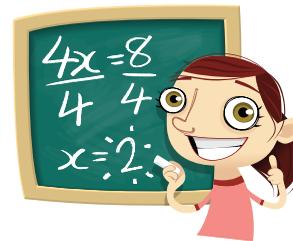
My partner works on the problem and shows her thinking.



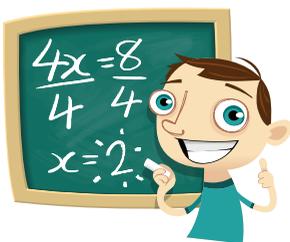
I share my thinking with my partner.



My partner shares her thinking with me.



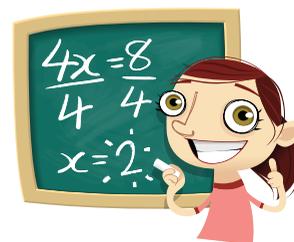
I show my partner's thinking.



I see that you ...

My partner shows my thinking.

I see that you ...



We compare our thinking and explain how they are similar and different.

In my solution ...



In your solution ...



Our thinking is similar because ...

Our thinking is different because ...

	<b>Prompt Starters</b>	<b>Response Starters</b>
<b>Explain &amp; Justify</b>	<ul style="list-style-type: none"> <li>• How did you approach the problem?</li> <li>• What did you do first?</li> <li>• Why did you...?</li> <li>• Can you explain how...</li> <li>• In your solution you... Why?</li> <li>• What does ... represent?</li> <li>• Tell me more about...</li> </ul>	<ul style="list-style-type: none"> <li>• First, I...</li> <li>• Next, I...</li> <li>• Then, I...</li> <li>• I used ... to show...</li> <li>• The reason I ... is because...</li> <li>• The ... represents...</li> <li>• I would like to add...</li> </ul>
<b>Compare</b>	<ul style="list-style-type: none"> <li>• How are our solutions alike and different?</li> <li>• How would you describe each solution?</li> <li>• How is my thinking similar to yours?</li> <li>• How is my thinking different from yours?</li> <li>• What is one connection you can make?</li> </ul>	<ul style="list-style-type: none"> <li>• Our solutions are alike because...</li> <li>• Our solutions are different because...</li> <li>• In my solution...</li> <li>• In your solution...</li> <li>• Our thinking is similar because...</li> <li>• Our thinking is different because...</li> <li>• One connection is...</li> </ul>

## SOAR SHARE & COMPARE

Name: \_\_\_\_\_ Partner's Name: \_\_\_\_\_ Date: \_\_\_\_\_

<input type="checkbox"/> MP1 – I made sense of the problem and did my best to solve it.	<input type="checkbox"/> I asked clarifying questions about my partner's idea.
<input type="checkbox"/> MP3 – I explained and justified my thinking using models, numbers, and words.	<input type="checkbox"/> I answered questions and gave examples to justify my solution.

<b>Problem</b>	
<b>My Solution Pathway</b>	<b>My Partner's Solution Pathway</b>
<b>Compare both solution pathways: How is my thinking similar to and/or different from my partner's thinking?</b>	

**The SOAR Teaching Frames® are based on a decade of research and help teachers, coaches, and administrators drive student learning by focusing on the most effective teaching practices aligned with the College and Career Readiness Standards. This manual is to be used in conjunction with a two-day Deep Dive designed to deepen understanding of the Mathematical Discussions Teaching Frame.**

"The SOAR teaching frames provide my teachers with support for planning rigorous and socioculturally relevant instruction for our diverse population of students. Problem solving and mathematical discourse have been a focus at our school and the SOAR Teaching Frames for Mathematics in particular are helping my teachers attend to the linguistic demands of the Common Core Standards for Mathematical Practice. More specifically, when considering the learning needs of English Learners, the frames help my teachers focus on teaching students the language required to comprehend and communicate their mathematical thinking effectively."

Michelle R. Staine, Assistant Principal  
Esperanza Elementary School  
Los Angeles Unified School District, Los Angeles, CA

"At the Center for Teacher Innovation we have used The Strategic Observation and Reflection (SOAR) Teaching Frames in training our reflective coaches to support teachers in implementing the California Content Standards. Our coaches are learning how to use the SOAR tools to identify and support a set of dynamic instructional moves effective teachers use to support student learning in Common Core classrooms that span grade levels and content areas. This suite of tools can be used to gather evidence and provide actionable feedback that will drive both teacher and student growth. The Frames are aligned to the California Standards for the Teaching Profession and have given our coaches an understanding of high-impact instructional practices and the capacity to coach beginning teachers in cultivating these practices."

Barbara Howard, Executive Director  
Center for Teacher Innovation  
Riverside County Office of Education, Riverside, CA

"SOAR is the convergence of long-term projects developed and implemented in California that position learning and professional growth at the center of teacher effectiveness and evaluation. The program provides a process for teachers and administrators to establish shared goals and objectives, instructional trust, and tools to calibrate their collective expectations of effective teaching. SOAR would be a significant help to any district looking for a professional growth model focused on improving the instructional and leadership practices of each educator."

Marcia G. Trott, Improving Teacher Quality State Grants Administrator  
California Department of Education, Sacramento, CA

"The SOAR frames have been the foundation for our current work in the district to transform our instructional practice. Not only do the frames provide us with common language and specific definitions of effective teaching practices, but they have also served as a detailed model for teachers, principals, and coaches. The frames have given our teachers specific strategies to implement and refine, which more actively engage students in their learning."

Ruben Reyes, Superintendent  
Robla School District, Sacramento, CA

[soarpractices.org](http://soarpractices.org)